

MATH 223 Spring
Assignment 11
Due: Monday, March 10

Reading

Read carefully Sections 4.3 “Directional Derivatives” in our text *Multivariable Calculus: A Linear Algebra Based Approach*.

Writing

Write out careful and complete solutions of Exercises 17 and 18 in Chapter 4 as well as Problems A, B, and C below.

Problem A: For each of these functions f find gradient $\nabla f(\mathbf{x})$ of f at a general point in the domain of f :

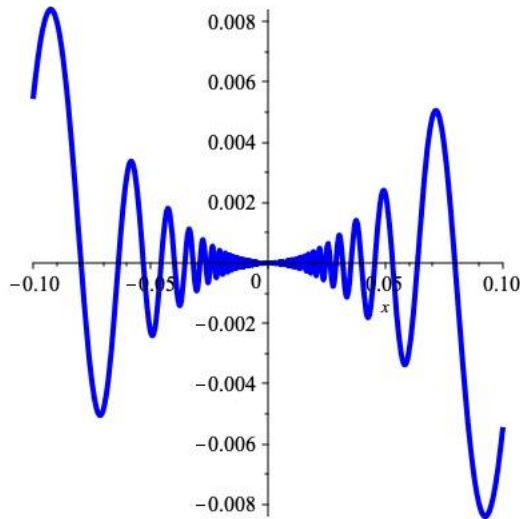
(1) $f(x, y) = 2x^3 - 3y^2$

(2) $f(x, y, z) = (5x - 7y)z$

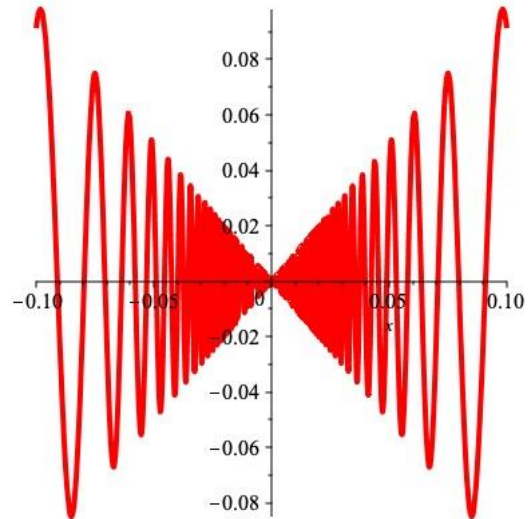
(3) $f(x_1, x_2, x_3) = \frac{x_1 x_3}{x_2}$

Problem B: Write an equation in terms of the coordinate variables (x, y, z) for the tangent hyperplane for $f(x, y, z) = 2x^2 - y^2 + 3z^2$ when $x = y = z = 1$.

Problem C: Let f be the real-valued function $f: \mathbb{R}^p \rightarrow \mathbb{R}$ defined by $f(\mathbf{x}) = |\mathbf{x}|^2 = \mathbf{x} \cdot \mathbf{x}$. If $p = 2$, prove that $\nabla f(\mathbf{x}) = 2\mathbf{x}$ for all \mathbf{x} in \mathbb{R}^p . Is this result true for other values of p ?



Graph of
 $f(x) = x^2 \sin\left(\frac{1}{x}\right)$



Graph of
 $f(x) = 2x \sin\left(\frac{1}{x}\right)$