

Multivariable Calculus: A Linear Algebra Approach  
**Errata and Additions**  
 August 27, 2023

Preface: Page xiii, last line: " Duncan McCabe, an undergraduate mathematics major at Middlebury, " should be "Duncan McCabe and Cody Matice, undergraduate mathematics majors at Middlebury, "

Chapter 1: Page 11: "We will usually write  $-(x)$ " should be "We will usually write  $-\mathbf{x}$ "

Page 15, line 6: Delete **Invertibility**

Page 15, 3 lines from bottom replace " $+c_3$ " with " $+c_3 - c_4$ "

**Page 17: Add Section on Mathematical Induction**

Chapter 2: Page 28, Example 2: "8 f/sec" should be "8 ft/sec"

Page 31: " $(kg)^{-2}$ " should be " $(\text{kg})^{-2}$ "

Page 34: In all of the equations replace lower case e with upper case E.

Page 41: Project 1: insert right parenthesis between  $\beta t$  and  $0 \leq t$

Chapter 3

Page 44, line 2: "An individual" should be "If an individual"

Page 47, Example 2:  $f_x$  should be  $f_x$ ,  $f_y$  should be  $f_y$  and  $f_z$  should be  $f_z$ .

Page 48, **Alternative Notation** replace "notation  $\frac{\partial f}{\partial x}$ " with "notation  $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right)$ "

Page 57: Second **Example 2** should be **Example 3**

Page 58: **Example 3** should be **Example 4**. Also " $\$D$ " should be " $\$D$ "

Page 58, 6 lines from bottom: Add colon (:) after **Solution**

Page 60, Exercise 20: " let " should be " Let "

Chapter 3: Page 62 , Exercise 7: Line 1:  $(x_4, x_5, 6)$  should be  $(x_4, x_5, x_6)$ .

Line 2: Replace existing text with. following:

(a) Determine  $f(10, 23, 18, 12, 30, 21)$ .

(b) Describe the set of points  $(x, y, z)$  in space such that  $f(1, 1, 1, x, y, z) = 0$ .

Chapter 4: Page 70: Line 6 of **Proof of (d)**:  $\mathcal{M}_2, \mathcal{M}_2$  should be  $\mathcal{M}_1, \mathcal{M}_2$

Page 71, Example 2::  $\cos xe^y$  should be  $\cos x + e^y$

Page 81, 3rd line of Example 3: Change "length of  $\mathbf{v}$  =" to "length of  $\mathbf{v}$ . which is "

Page 84, Line 2: delete period in "joining them as."

Page 86, Second line of Section 4.6:  $-1 \leq \cos \theta \leq 1$  should be  $-1 \leq \cos \theta \leq 1$

Page 89, Second Line of 4.6.2 Newton's Method:  $x - 2 = 0$  should be  $x^2 - 2 = 0$

Page 96, Exercise 17: Delete left parenthesis ( from "for  $(x \neq 0$

Page 98, Line 2: right after "vector - valued functions", add "in Exercises 30 - 32"

Page 99, Exercise 39: Change "1,00,000" to "1,000,000"

Chapter 5 : **Need to Number the Theorems.**

Page 101: First Line of 5.1.1: Change " **Theorem: The Little Chain Rule** " to " **Theorem 5.1.1 (The Little Chain Rule)** :"

Page 103: First Line of 5.1.2: Change " **Theorem: General Chain Rule** " to " **Theorem 5.1.2 (General Chain Rule)** :"

Page 104, Line 1: Insert colon (:) immediately after **Solution**

Page 107, second line after "so":  $= 9f(u)^2 + 12f_u^* f_v^* + 4(f_v^*)^2 + 4f(u)^2 - 12f_u^* f_v^* + 9f(v)^2$   
 should be  $= 9(f_u^*)^2 + 12f_u^* f_v^* + 4(f_v^*)^2 + 4(f_u^*)^2 - 12f_u^* f_v^* + 9(f_v^*)^2$

Page 109, Line 7 in Section 5.3: " $\frac{\partial f}{\partial x_1} \frac{\partial f}{\partial x_2}$ " should be " $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}$ "

Page 110: 4th line in Example 3: "The text " should be "The test "

Page 113: In paragraph "Let's look a bit": Change "differentiable function real-valued" to "differentiable real-valued"

Page 121, Example 11: Both instances of "\$C" should be "\$C"

Page 121, 2 line from bottom: " $3xx$ " should be " $3x$ "

Page 124, First Definition: "*Hessian Matrix*" should be "**Hessian Matrix**"

Page 124, Second Definition: "*Positive-Definite Matrix*" should be "**Positive-Definite Matrix**" and "*positive semi-definite*" should be "**positive semi-definite**"

Page 129, in Example 2, change "You can" to "We can "

Page 131, Sentence beginning "Figure 5/17 illustrates ": insert period at end of sentence.

Page 139, Exercise 37. Delete period (.) in first line.

Page 139, Exercise 42, last line: Add right parenthesis so  $(\mathbf{c}, f(\mathbf{c}))$  becomes  $(\mathbf{c}, f(\mathbf{c}))$

Page 139, Exercise 48: add:

(a): Show that  $f(x, y) = 4x^2 + 8xy + 5y^2$  is a positive-definite function.

(b) Show that  $g(x, y) = -x^2 + 2xy - 3y^2$  is a negative-definite function

Page 141, Exercise 63: Change "\$T" to "\$T"

Page 142, Exercise 64: Change "\$p" to "\$p", "\$q" to "\$q", and "\$B" to "\$B"

Page 142, Exercise 65: Change "\$C" to "\$C"

## Chapter 6

Page 158: italicize  $x$  in **x-simple** and the  $y$  in **y-simple**

Page 160, 7 lines from bottom: delete period in "such that."

Page 161, 8 lines from bottom: "with mesh less than " should be "with mesh less than "

Page 162, **Proof of (2)**: Change "use" to "Use"

Page 163, Line 7: Insert line break just before **Partition Theorem**

Page 167, Line 3; Insert equality sign (=) between  $\frac{1}{2}(u-1)^{-1}$  and  $\frac{1}{2\sqrt{u-1}}$ .

Page 174, Line 3 of **Solution**: Add colon after *Method A*

Page 175, 8 lines from bottom: "as  $as \rightarrow 0$ " should be "as  $a \rightarrow 0$ "

Page 175, 3 lines from bottom " $\int_D f DA$ " should be " $\int_D f dA$ "

Page 182, Exercise 8: " $x - axis$ " should be " $x$ -axis"

Page 185, Exercise 32: Add at the start: "Determine the convergence or divergence of the integral"

Page 185,k Exercise 35b: Formula at end of the line should read

$$\frac{1}{\lambda} [\lambda^2 - 1 + e^{-3\lambda}(11 + 3\lambda)]$$

## Chapter 7:

Page 191, About 5 lines from bottom: Equation with *Work* =: "Work" should not be italicized

Page 196, 9 lines from bottom: " $0 \leq s11$ " should be " $0 \leq s \leq 1$ "

Page 201, 4th line in Solution of Example 6: " $\cos^t$ " should be " $\cos^2 t$ "

Page 201, 8th line in Solution of Example 6:  $\mathcal{L}(\gamma) = \int_0^{\pi/2} \int |\mathbf{g}'(t)| dt =$  should be  $\mathcal{L}(\gamma) = \int_0^{\pi/2} |\mathbf{g}'(t)| dt =$

Page 201, 11th line in Solution of Example 6:  $\mathcal{L}(\gamma) = \int_0^{\pi/2} \int |\mathbf{g}'(t)| dt =$  should be  $\mathcal{L}(\gamma) = \int_0^{\pi/2} |\mathbf{g}'(t)| dt =$

Page 202, Example 7: "  $4\pi$  " should be "  $4\pi$  "

Page 205, 7 lines from bottom: "straight forward" should be "straightforward"

Page 211, 2nd line of proof of Theorem 6.2: "  $\mathbf{T}' = \frac{d\mathbf{T}}{t}$  " should be "  $\mathbf{T}' = \frac{d\mathbf{T}}{dt}$  "

Page 220, Exercise 12: "  $2xx^2$  " should be "  $2x^2$  "

Page 220, Exercise 14: "  $\mathbf{g}$  " should be "  $\mathbf{g}(t)$  "

Page 220, Exercise 15: "  $\mathbf{h}s$  " should be "  $\mathbf{h}(t)$  "

Page 222, Exercise 26: "  $\cosh$  " should be "  $\sinh$  "

Page 222, Exercise 28: " **y-axis** " should be "  $y$ -axis "

Page 222, Exercise 30: Sketch the solid obtained by revolving the graph of  $y = 4\sqrt[3]{x}$  from (8,8) to (27, 12) around the  $y$ -axis and determine its surface area.

Page 223, Replace Exercise 41 of Chapter. 7 with:

For each of the following, sketch the curve with the given equation and determine its curvature:

(a)  $y = x^2$

(b)  $y = x^3$

(c)  $y = x^4$

(d)  $y = x^5$

(e)  $y = x^{-1}, x > 0$

(f)  $y = x^{-2}, x > 0$

(g)  $y = \ln x, x > 0$

Page 224, Exercise 48(b): "  $-\frac{2A}{5}$  " should be "  $\frac{2A}{5}$  "

Page 224, Exercise 49: "  $7x$  " should be "  $-7x$  "

Page 224, Exercise 50: "  $-x + 3y$  " should be "  $4x + y$  "

Page 225, Exercise 55(e): "  $v_1v_2V_2$  " should be "  $v_1v_2v_3$  "

Page 226, **Art Gallery Problem**: "  $\lfloor 3 \rfloor = \lfloor 4 \rfloor = \lfloor 5 \rfloor = 1$ , but  $\lfloor 6 \rfloor = 2$ . " should be "  $\lfloor 3/3 \rfloor = \lfloor 4/3 \rfloor = \lfloor 5/3 \rfloor = 1$ , but  $\lfloor 6/3 \rfloor = 2$ . "

Page 230, Add colon after **Example 3**

#### Chapter 8:

Page 229, 2nd line from bottom: "For points above this line,  $\operatorname{div} \mathbf{F}$  is positive " should be "For points below this line,  $\operatorname{div} \mathbf{F}$  is positive "

Page 248: Line. 1: "  $X^2$  " should be "  $x_2$  "

Page 258, 9 lines from bottom: "  $dx dy dx$  " should be "  $dx dy dz$  "

Page 276. Exercise 21. (a) and (b): "  $- \leq t$  " should be "  $0 \leq t$  "

Page 275, Exercise 5: "  $m \times n$  " should be "  $n \times n$  "

Page 276, Exercise 18: "  $(y + 2x, x + 3x, 2x + 3y)$  " should be "  $(y + 2x, x + 3z, 2z + 3y)$  "

Page 276, Exercise 21(a) and (b) "  $- \leq t$  " should be "  $0 \leq t$  "

Page 277, Exercise 29(d), 3rd line: "  $g(\sigma(S, t))$  " should be "  $f(\sigma(s, t))$  "

Page 277, Exercise 31: The vector field should be  $\mathbf{F}(x, y, z) = (x, 2y, 3z)$ .

#### Chapter 9:

Page 290, Line 2: " *partialR* " should be "  $\partial R$  "

Page 291, Exercises 3, 4, 5: " *gamma* " should be "  $\gamma$  "

Chapter 9:

Page 290, 2nd line of Proof of Theorem: The dash (-) should be an equal sign (=)

Page 291: Replace Exercises 1 - 5 with:

1. Find  $\int_{\gamma} \omega$  if  $\omega = \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy$  and  $\gamma$  is the circle of radius 2, centered at the origin and traced out counterclockwise.
2. Find  $\int_{\gamma} \omega$  if  $\omega = x dx + dy$  and  $\gamma$  the graph of  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  traced out counterclockwise.
3. Find  $\int_{\gamma} \omega$  if  $\omega = y dx + x dy$  and  $\gamma$  has the parametrization  $g(t) = (\cos t, \sin t, t), 0 \leq t \leq 2\pi$ .
4. Find  $\int_{\gamma} \omega$  if  $\omega = (x+y) dx + zy^2 dy + (x-z) dz$  and  $\gamma$  in the line segment from (1, 2, 3) to (4, -3, 7).
5. Find  $\int_{\gamma} \omega$  if  $\omega = (x^2 + yz) dx + z dy + (y-x) dz$  and  $\gamma$  has parametrization  $g(t) = (2t+3, 3t-2, t+4), 0 \leq t \leq 2$ .

**IF POSSIBLE, ADD THE FOLLOWING AT END OF SECTION 1.2.4 BEFORE SECTION 1.3:**

**Example:** Suppose  $\lambda \neq \mu$  are distinct eigenvalues of the matrix  $A$  with corresponding eigenvectors  $\mathbf{v}$  and  $\mathbf{u}$ . Thus  $\mathbf{v}$  and  $\mathbf{u}$  are nonzero vectors with  $A\mathbf{v} = \lambda\mathbf{v}$  and  $A\mathbf{u} = \mu\mathbf{u}$ . Show that  $\{\mathbf{v}, \mathbf{u}\}$  is a linearly independent set.

*Solution:* Suppose  $c$  and  $d$  are constants such that

$$(*) \quad c\mathbf{v} + d\mathbf{u} = \mathbf{0}.$$

We need to show that both  $c$  and  $d$  must be 0. Let's multiply both sides of (\*) by the matrix  $A$ :

$$A(c\mathbf{v} + d\mathbf{u}) = A(c\mathbf{v}) + A(d\mathbf{u}) = cA\mathbf{v} + dA\mathbf{u} = c\lambda\mathbf{v} + d\mu\mathbf{u} = A\mathbf{0} = \mathbf{0}.$$

Thus we have

$$(**) \quad c\lambda\mathbf{v} + d\mu\mathbf{u} = \mathbf{0}.$$

If we multiply (\*) by  $\lambda$ , we obtain

$$(***) \quad c\lambda\mathbf{v} + d\lambda\mathbf{u} = \mathbf{0}.$$

Subtract equation (\*\*\*) from (\*\*) to obtain

$$d(\mu - \lambda)\mathbf{u} = \mathbf{0},$$

but  $\mu - \lambda$  is a nonzero number since the eigenvalues are distinct. Hence  $d\mathbf{u} = \mathbf{0}$  which implies  $d = 0$  because  $\mathbf{u}$  is a nonzero vector. Equation (\*) reduces to  $c\mathbf{v} = \mathbf{0}$  which also implies  $c = 0$  by the nonzero character of the vector  $\mathbf{v}$ .

**Add a New Exercise 24 for Chapter 1:**

**Exercise:** Let  $\lambda_1, \lambda_2, \lambda_3$  be three distinct eigenvalues of the same matrix  $A$  with corresponding eigenvectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ . Prove that the set of these eigenvectors is linearly independent.