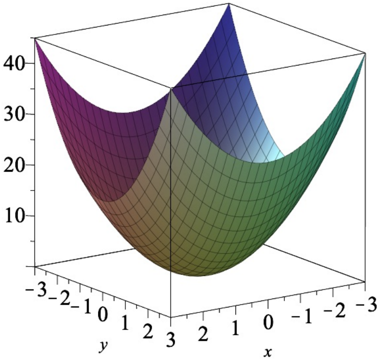
MATH 223: Multivariable Calculus



Class 5: February 19, 2025

Notes on Assignment 3

Assignment 4

Simple Animation in MATLAB

Simple Animation in MATLAB

Animate Ellipse $(3\cos t, 5\sin t), 0 \le t \le 2\pi$

```
axis([-6 6 -6 6])
hold on
N = 20;
for i = 1:N
    t = linspace(0,i*2*pi/N );
    pause(0.2)
    plot( 3* cos(t), 5 *sin(t),'b', 'LineWidth',3)
end
```

Animate Strange Curve $(\cos 3t, \sin 5t), 0 \le t \le 2\pi$

```
axis([-2 2 -2 2])
hold on
N = 20;
for i = 1:N
    t = linspace(0,i*2*pi/N );
    pause(0.2)
    plot( cos(3*t), sin(5*t),'r', 'LineWidth',3)
end
```

Animate Helix $(\cos t, \sin t, t)$ for $0 \le t \le 6\pi$

```
N = 30;
for i = 1:N
    t = linspace(0, i* 6 * pi/N);
    pause(0.3)
    plot3( cos(t), sin(t),t, 'LineWidth',3)
    xlim(([-1 1])); ylim([-1 1]); zlim([0 20]);
end
xlabel('x');
ylabel('y');
zlabel('z');
```

Major Topic: Real-Valued Functions of Vectors

$$f : \mathcal{R}^n \to \mathcal{R}^1$$

(I) Defining and Using Derivatives
Chapters 3 – 5

(II) Integrals
Chapters 6 – 7

Initial Focus: Geometry of Such Functions

Some Useful Pictures

(A) $f : \mathcal{R}^2 \to \mathcal{R}^1 \quad z = f(x, y)$

Image: Interval of Real Numbers

Domain: Region in Plane

Graph: Surface in $\mathcal{R}^3$

In General, graph $f : \mathcal{R}^n \to .\mathcal{R}^1$ is the set of points of the form $(\mathbf{x}, f(\mathbf{x}))$, an n-dimensional surface in $n+1$-dimensional space.

(B) Contours

Level Curves for $f : \mathcal{R}^2 \to .\mathcal{R}^1$

Level Surfaces for $f : \mathcal{R}^3 \to .\mathcal{R}^1$

C) Cross-Sections

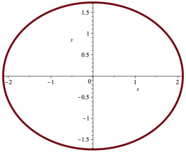
Fix one variable

Example: $f(x,y) = 2x^2 + 3y^2$ or $z = 2x^2 + 3y^2$.

Observations:

1. $z \geq 0$ for all x, y; $z =$ only at (0,0)
2. Hold z fixed, say. $z = z_0 > 0$

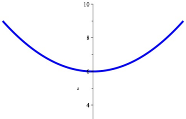
$$2x^2 + 3y^2 = z_0$$

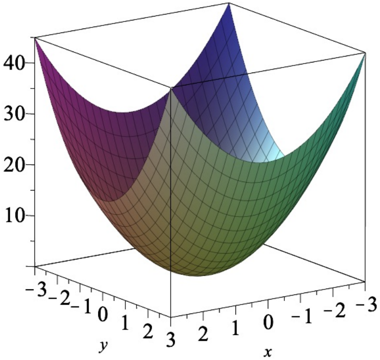


3. Hold x fixed: $x = x_0$

$$2x_0^2 + 3y^2 = z$$

Parabola in (y,z)-plane





Level Curves

Given: output k (some constant)

Find: All inputs Which Produce That Output

Examples

Isotherms

Isobars

Isoclines

Indifference Curves

FUNDAMENTAL DIFFERENCE BETWEEN DOMAIN BEING SUBSET OF $\mathcal{R}^1$ AND SUBSET OF $\mathcal{R}^n, n > 1$:

Implications for Continuity, Derivative, Integral

These all depend on **LIMIT**:

$\lim_{x \to a} f(x)$	$\lim_{\mathbf{x} \to \mathbf{a}} f(x)$
$\mathcal{R}^1$	$\mathcal{R}^n$
2 ways to approach	Infinitely Many Ways

Example:

$$f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Approach along line $y = mx$:

$$f(x,mx) = \frac{x(mx)}{x^2+2(mx)^2} = \frac{mx^2}{x^2+2m^2x^2} = \frac{mx^2}{x^2(1+2m^2)} = \frac{m}{1+2m^2}$$

m	$\frac{m}{1+2m^2}$
1	$\frac{1}{1+2} = \frac{1}{3}$
2	$\frac{2}{1+8} = \frac{2}{9}$
-1	$\frac{-1}{1+2} = -\frac{1}{3}$

What are the possible values of $g(m) = \frac{m}{1+2m^2}$?

$g'(m) = \frac{1-2m^2}{(1+2m^2)^2}$ and $g''(m) = \frac{4m(2m^2-3)}{(1+2m^2)^3}$

- $g(m) \to 0$ as $m \to \pm\infty$
- $g'(m) > 0$ for $\sqrt{2}/2 \le m \le \sqrt{2}/2$
- Point of Inflection at $\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$