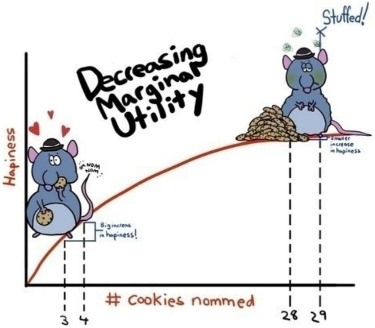
MATH 223: Multivariable Calculus



Class 9

Friday, February 28, 2025

- Notes on Assignment 8
- Assignment 9

Announcements

Exam 1: Next Monday, 7 PM -
No Time Limit

Last Name A to J: Warner 100
Last Name K to Z: Warner 101

No Books, Computers, Smartphones, etc.

One Page of Notes OK

Focus on Chapters 2 and 3

Tangent Planes To Surfaces

(I) $f : \mathcal{R}^2 \to \mathcal{R}^1$**, a a point in** $\mathcal{R}^2$

Tangent plane to graph of f at $(\mathbf{a}, f(\mathbf{a}))$:

$$T(\mathbf{x}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$$

(II): $f : \mathcal{R}^2 \to \mathcal{R}^3$

$$\sigma(s,t) = (f(s,t), g(s,t), h(s,t))$$

$$\sigma_s(s,t) = (f_s, g_s, h_s) \text{ and. } \sigma_t(s,t) = (f_t, g_t, h_t)$$

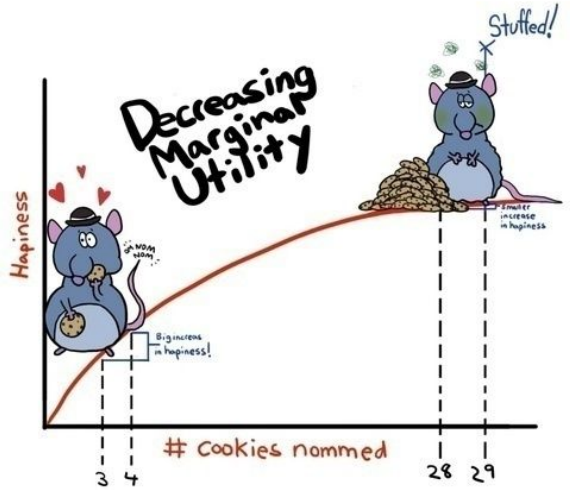
Tangent Plane at $\sigma(\mathbf{a})$:

$$\sigma(\mathbf{a}) + (s,t)\begin{pmatrix} f_s(\mathbf{a}) & g_s(\mathbf{a}) & h_s(\mathbf{a}) \\ f_t(\mathbf{a}) & g_t(\mathbf{a}) & h_t(\mathbf{a}) \end{pmatrix}$$

Note: $1 \times 3 + (1 \times 2)(2 \times 3)$

Writing vectors vertically: $\sigma = \begin{pmatrix} f \\ g \\ h \end{pmatrix}, \sigma' = \begin{pmatrix} f' \\ g' \\ h' \end{pmatrix}$

Tangent Plane: $T\begin{pmatrix} s \\ t \end{pmatrix} = \sigma(\mathbf{a}) + \sigma'(\mathbf{a})\begin{pmatrix} s \\ t \end{pmatrix}$



Decreasing Marginal Utility
Stuffed!
Hapiness
NOM NOM
Big increas
in hapiness!
Smaller
increase
in hapiness
cookies nommed
3
4
28
29

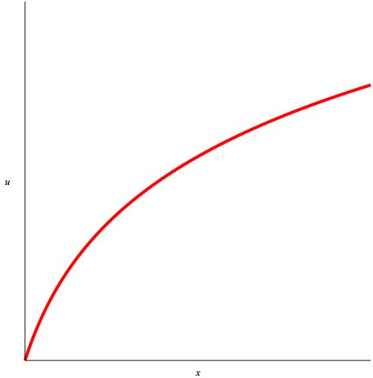
Utility

Utility = happiness, satisfaction, pleasure, usefulness

$u(x), x \geq 0$

Typical Assumptions: u is increasing, concave down function

("decreasing returns to scale")



<u>Example</u>: $u(x) = x^{1/3}$ so $u'(x) = \frac{1}{3x^{2/3}}, u''(x) = -\frac{2}{9}x^{-5/3}$

Example: 2 Goods with $u(x, y) = \sqrt[3]{xy}$
Each unit of x costs \$35 and each unit of y costs \$80
We have \$D to spend: Budget Constraint: $35x + 80y = D$
Goal: Maximize Utility:

$$80y = D - 35x \text{ so } y = \frac{D - 35x}{80}$$

$$u(x, y) = f(x) = \sqrt[3]{\frac{x(D - 35x)}{80}}$$

f is maximized when $\frac{x(D-35x)}{80}$ is maximized.
$G(x) = x(D - 35x). = Dx - 35x^2.$ has $G'(x) = D - 70x$ and $G''(xx) = -70$ Hence there is a maximum when $x = D/70$
Then $y = \frac{D-35(D/70)}{80} = D/160$

Clairaut's Theorem on Equality of Mixed Partials

If f_{xy} and f_{yx} are continuous at $\mathbf{a}$, then $f_{xy}(\mathbf{a}) = f_{yx}(\mathbf{a})$