# VECTORS IN TWO DIMENSIONS

| | | |
|---|---|---|
| scalar quantity | scalar | directed line segment |
| initial point | terminal point | magnitude |
| equivalent vectors | equal vectors | velocity vector |
| force vector | displacement | sum of vectors |
| resultant force | scalar multiple | position vector |
| components of vector | $\mathbf{a} = ( a_1,a_2 )$ | |

*Notation:* $\mathbb{R}^2$ is the set of all vectors ( $x,y$ ) where $x$ and $y$ are real numbers.

*Definition:* The **magnitude** $\|\mathbf{a}\|$ of the vector $\mathbf{a} = ( a_1,a_2 )$ is $\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2}$

*Definition:* (Addition of Vectors): $( a_1,a_2 ) + ( b_1,b_2 ) = ( a_1 + b_1, a_2 + b_2 )$

*Definition:* (Scalar Multiple of a Vector): $c ( a_1,a_2 ) = ( ca_1,ca_2 )$

*Definition:* The **zero vector** $\mathbf{0}$ is $\mathbf{O} = ( 0,0 )$,
and the **negative of a** is $-\mathbf{a} = -( a_1,a_2 ) = ( -a_1, -a_2 )$

*Theorem*: The follow properties hold for vectors **a, b, w** and all scalars $c$ and $d$:

| | | |
|---|---|---|
| $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ | $\mathbf{a} + (\mathbf{b}+\mathbf{w}) = (\mathbf{a} + \mathbf{b}) + \mathbf{w}$ | $\mathbf{a} + \mathbf{0} = \mathbf{a}$ |
| $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$ | $c(\mathbf{a}+\mathbf{b}) = c\mathbf{a} + c\mathbf{b}$ | $(c+d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$ |
| $(cd)\mathbf{a} = c(d\mathbf{a}) = d(c\mathbf{a})$ | $1\mathbf{a} = \mathbf{a}$ | $0\mathbf{a} = \mathbf{0} = c\mathbf{0}$ |

*Definition:* If $\mathbf{a} = (a_1,a_2)$ and $\mathbf{b} = (b_1,b_2)$, the **difference a - b** is **a + (-b).**

*Theorem*: If $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are any two points, then the vector **a** in $\mathbb{R}^2$ that corresponds to $P_1P_2$ is $\mathbf{a} = ( x_2 - x_1, y_2 - y_1 )$.

*Definition:* Nonzero vectors **a** and **b** in $\mathbb{R}^2$ have
(i) the **same direction** if $\mathbf{b} = c\mathbf{a}$ for some positive scalar $c > 0$.
(ii) the **opposite direction** if $\mathbf{b} = c\mathbf{a}$ for some negative scalar $c < 0$.
The vectors **a** and **b** are **parallel** if $\mathbf{b} = c\mathbf{a}$ for some scalar $c$

*Theorem*: If **a** is a vector and $c$ is a scalar, then $\| c\,\mathbf{a} \| = |c|\,\| \mathbf{a} \|$

*Definition:* $\mathbf{i} = ( 1,0 )$, $\mathbf{j} = ( 0,1 )$

*Theorem:* If $\mathbf{a} = ( a_1,a_2 )$, then $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$

*Definition:* If $\mathbf{a} = ( a_1,a_2 )$, then $a_1$ is the **horizontal component** of **a** and $a_2$ is the **vertical component** of **a.**

# VECTORS IN THREE DIMENSIONS

| | | |
|---|---|---|
| ordered triple | *xy*-plane | *x*- coordinate |
| equality of triples | *yz*-plane | y- coordinate |
| origin | *xz*-plane | *z*-coordinate |
| right-handed coordinate system | rectangular coordinate system | *xyz*-coordinate system |

*Distance Formula:* The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

*Midpoint Formula:* The midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

*Definition:* The **graph of an equation** in three variables *x, y*, and *z* is the set of all points *P(a,b,c)* in a rectangular coordinate system such that the ordered triple *(a,b,c)* is a solution of the equation. The graph of such an equation is called a **surface**.

*Theorem:* An equation of a sphere of radius *r* and center $P_0(x_0, y_0, z_0)$ is

$$(x - x_0)^2 + (y - y)^2 + (z - z_0)^2 = r^2$$

*Definition:* $\mathbb{R}^3$ is the set of all vectors (*x,y,z* ) where *x, y* and *z* are real numbers.

*Definition:* In $\mathbb{R}^3$, **i** = ( 1,0,0 ), **j** = ( 0,1,0 ), and **k** = ( 0,0, 1).

Note: We may write the vector **a** = $( a_1, a_2, a_3 )$ as **a** = $a_1$**i** + $a_2$**j** + $a_3$**k**