Math 223: Multivariable Calculus
Exam 1 Review

This is a list of topics for our first exam. This list is not necessarily exhaustive, but it covers the main ideas we have seen since the beginning of the semester.

Remember that the main objective of the exam is to give you a chance to review the things we have learned in our class and to help solidify your mental connections between them. When studying, you can use homework problems, our extended problem list, and/or other problems from the textbook as practice. If there is a particular concept or area that you find challenging, you might consider focusing your attention on problems related to that topic.

To review the theory, you can look at the theorems in the book; review the online videos, notes, and homework to see how we applied the theorems; review how our various formulas were derived; and, for any of the homework problems, consider how you would justify your computational answer. I won't ask you to re-derive any of the formulas we have used, but knowing for yourself where they came from can really help to highlight the underlying concepts so that you will be able to use them in increasingly fluid and flexible ways.

- Points and Vectors in $\mathbb{R}^n$
  - Vectors versus points in $\mathbb{R}^n$
  - Algebraic and geometric properties of vectors in $\mathbb{R}^2$, $\mathbb{R}^3$, and $\mathbb{R}^n$
  - Dot and cross products: how to compute and properties of each
  - Lengths of and angles between vectors
  - Orthogonal projection of vectors
  - Planes and lines in $\mathbb{R}^n$: how to compute and express. The equation of a plane in $\mathbb{R}^3$. Distance between points and lines/planes
  - Rectangular, polar, cylindrical, and spherical coordinate systems: how to convert between coordinate systems and how to express surfaces and solids in $\mathbb{R}^3$ using the different coordinate systems

- Calculus with Multivariable Functions
  - Multivariable functions: domain, codomain, range
  - Graphing functions $f : \mathbb{R}^2 \to \mathbb{R}$: using level curves and/or cross-sections

– Level surface of a function $F : \mathbb{R}^3 \to \mathbb{R}$. Realizing the graph of $f(x, y)$ as a level surface of a function $F : \mathbb{R}^3 \to \mathbb{R}$

– General concept of level set of scalar-valued function $f : \mathbb{R}^n \to \mathbb{R}$

– Drawing/recognizing familiar quadric surfaces

– Limits and continuity of multivariable functions

– Partial derivatives: computing and interpreting. Higher-order partials

– Differentiability of $f(x, y)$. Tangent planes to graphs

– Total derivative of $f : \mathbb{R}^n \to \mathbb{R}^m$: definition and interpretation. Simple computation of differentiability and derivative