Math 223 - Multivariable Calculus Name:

Practice Exam 2

Please be sure to neatly **show and explain all of your work** and clearly label your answers. Other than your index card, this is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points .
1	/12
2	/10
3	/10
4	/12
5	/12
6	/12
7	/12
Total	/80

1. Please compute the following. Show all work.

   (a) What is the arc length of the curve $\bar{r}(t) = (2t, t^2, \frac{1}{3}t^3)$, $0 \le t \le 1$?

   (b) If $\bar{F}(x, y, z) = (xz^2, x^2e^{yz}, x^2y^3z^4)$, please compute $\text{curl}\bar{F}$.

2. Consider the function $f(x, y) = x^2 + 3xy$. Let $\bar{a} = (0, 1)$.

   (a) Starting at the point $\bar{a}$, in what direction(s) is the directional derivative of f the greatest? Please give your answer in the form of (a) vector(s) and justify your response.

   (b) Starting at the point $\bar{a}$, in what direction(s) is the directional derivative of f equal to 0? Please give your answer in the form of (a) vector(s) and justify your response.

   (c) What is the rate of change of f in the direction of a unit vector that points halfway between the direction of greatest change and the direction of 0 change? Please show all work and explain your reasoning.

3. Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is a scalar-valued function on $\mathbb{R}^3$ and suppose $\bar{r} : \mathbb{R} \to \mathbb{R}^3$ defines a curve in $\mathbb{R}^3$, with $\bar{r}(0) = \bar{a}$. Let the component functions of $\bar{r}(t)$ be $(x(t), y(t), z(t))$.

   Since f is defined on all of $\mathbb{R}^3$, it makes sense to consider $f \circ \bar{r}$. (We often think of this as the restriction of f to the curve defined by $\bar{r}$.)

   Notice that $f \circ \bar{r} : \mathbb{R} \to \mathbb{R}$.

   Show that
$$\frac{d}{dt}(f \circ \bar{r})_{|t=0} = \nabla f(\bar{a}) \cdot \bar{r}'(0).$$

   Be sure to justify each step that you make.

   (Hint: as a first step, it is helpful to note that for any function $h : \mathbb{R} \to \mathbb{R}$, the notation $\frac{d}{dt}(h)_{|t=b}$ means the same thing as $Dh(b)$.)

4. Consider $f(x, y) = 2x^2 + 3xy + 4x - 6y$

(a) Please find all critical points of $f(x, y)$. Show all work.

(b) Choose a critical point from part (b) and classify it as a local maximum, local minimum, or neither.

5. Find the absolute minimum value of

$$f(x, y) = x^2 + 2y^2$$

on the circle $x^2 + y^2 = 1$. Show all work and clearly justify why you know you have found a maximum.

6. Compute $\iint_R f(x,y)\,dA$ where $f(x,y) = x^2y$ and R is the region in the xy-plane bounded by the curves $y = 0$, $x = 1$, and $y = 2x^2$. Show all work.

7. Observe that the point $(1, 0, 0)$ lies on both the surface $2x^2+y^2+z^2 = 2$ and the curve $\bar{r}(t) = (t^2, t - 1, 2t - 2)$. (How do you know?)

   At the point $(1, 0, 0)$, at what angle does the tangent line to $\bar{r}(t)$ intersect the normal line to the surface? Show all work and explain your reasoning.