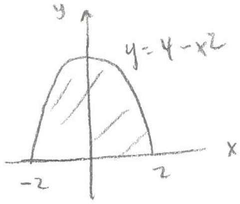
5.2

3a.



$$\int_{-2}^{2}\int_{0}^{4-x^2} x^3 \, dy\,dx$$

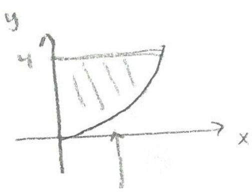
$$= \int_{-2}^{2} yx^3 \Big|_{0}^{4-x^2} dx$$

$$= \int_{-2}^{2} (4-x^2)x^3 - 0 \, dx$$

$$= \int_{-2}^{2} 4x^3 - x^5 \, dx = x^4 - \frac{1}{6}x^6 \Big|_{-2}^{2} = \left[2^4 - \frac{1}{6}(2^6)\right] - \left[(-2)^4 - \frac{1}{6}(-2)^6\right]$$

$$= \boxed{0.}$$

9.



$x = 2\sqrt{y}$

$y = \frac{x^2}{4}$

$$\int_{0}^{4}\int_{0}^{2\sqrt{y}} x\sin(y^2)\,dx\,dy$$

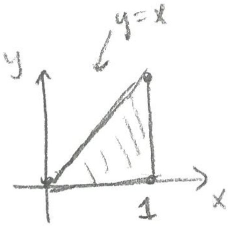
$$= \int_{0}^{4} \frac{x^2}{2}\sin y^2 \Big|_{0}^{2\sqrt{y}} dy$$

$$= \int_{0}^{4} \frac{4y}{2}\sin(y^2)\,dy = -\cos(y^2)\Big|_{0}^{4} = \boxed{-\cos(16) + 1}$$

$u = y^2$

$\frac{du}{dy} = 2y \rightsquigarrow dy = \frac{du}{2y}$

21.



$$\int_{0}^{1}\int_{0}^{x} e^{x^2}\,dy\,dx$$

NOTE: the other order would be impossible!

$$\int_{0}^{1}\int_{y}^{1} e^{x^2}\,dx\,dy$$

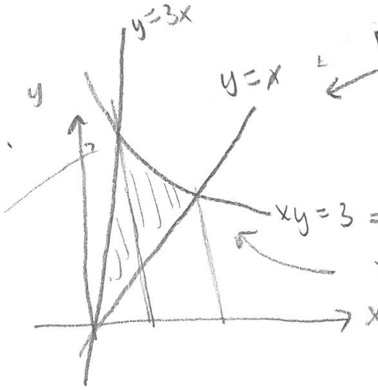
cant integrate e^{x^2}!

$$= \int_{0}^{1} ye^{x^2}\Big|_{0}^{x} dx$$

$$= \int_{0}^{1} xe^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_{0}^{1} = \boxed{\frac{1}{2}e - \frac{1}{2}}$$

$u = x^2$

24.



NOTE: either order of integration will require a sum of 2 integrals.

$xy = 3 \Rightarrow y = 3/x$

$x = \frac{3}{x} \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}$

$3x = \frac{3}{x}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = 1$

$$\int_0^1 \int_x^{3x} (x^2+y^2)\,dy\,dx + \int_1^{\sqrt{3}} \int_x^{3/x} x^2+y^2\,dy\,dx$$

$$= \int_0^1 x^2y + \frac{1}{3}y^3 \Big|_x^{3x} dx + \int_1^{\sqrt{3}} x^2y + \frac{1}{3}y^3 \Big|_x^{3/x} dx$$

$$= \int_0^1 (3x^3 + 9x^3 - x^3 - \frac{1}{3}x^3)\,dx + \int_1^{\sqrt{3}} (3x + \frac{9}{x^3} - x^3 - \frac{1}{3}x^3)\,dx$$

$$= \int_0^1 \frac{32}{3}x^3\,dx + \int_1^{\sqrt{3}} (3x + \frac{9}{x^3} - \frac{4}{3}x^3)\,dx$$

$$= \frac{8}{3}x^4 \Big|_0^1 + \left[\frac{3}{2}x^2 - \frac{9}{2x^2} - \frac{1}{3}x^4\right]_1^{\sqrt{3}} = \frac{8}{3} + \left[\frac{9}{2} - \frac{3}{2} - 3\right] - \left[\frac{3}{2} - \frac{9}{2} - \frac{1}{3}\right]$$

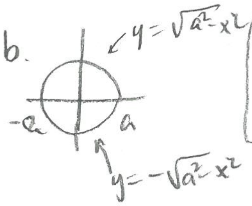
$$= \frac{8}{3} + 3 + \frac{1}{3} = \boxed{6.}$$

28. $\iint_D 1\,dA$ = volume under graph of 1 above region D

$= \underbrace{1}_{\text{height}} \underbrace{(\text{area } D)}_{\text{area base}}$

= area D.

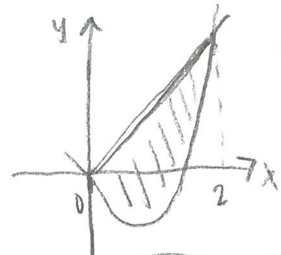
b.



$$\boxed{\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 1\,dy\,dx}$$

(probably trig sub!)

38.



$y = x = x^2 - x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$$\boxed{\int_0^2 \int_{x^2-x}^{x} x^2 + 6y^2\,dy\,dx}$$