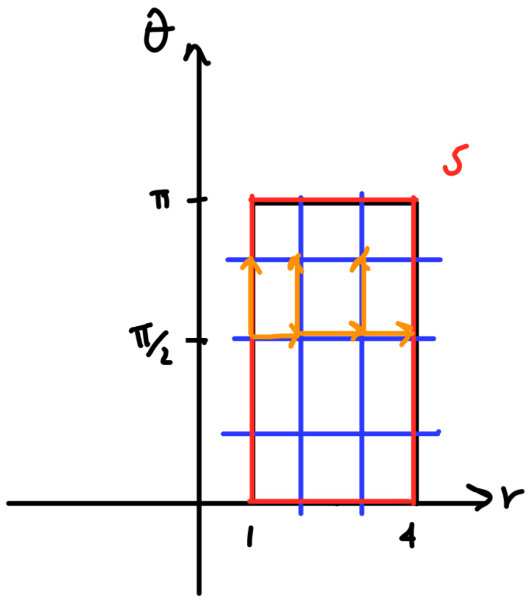
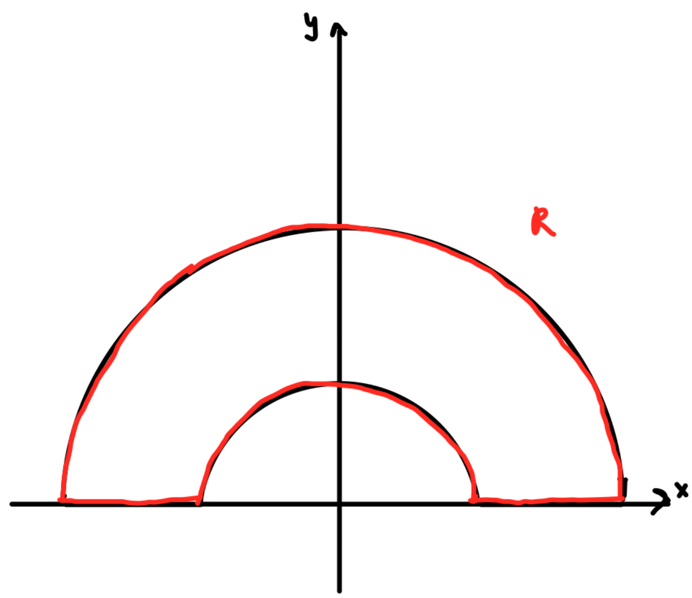
Math 223: Multivariable Calculus
The Total Derivative

Consider the polar coordinate transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(r, \theta) = (r\cos\theta, r\sin\theta)$.

In this worksheet, we will explore the total derivative DT of T, evaluated at some representative points in the domain space S of T.

Recall that $DT(r_0, \theta_0)$ maps vectors based at (r_0, θ_0) to vectors based at $T(r_0, \theta_0)$.





1. For each point (r_0, θ_0) equal to $(1, \frac{\pi}{2})$, $(2, \frac{\pi}{2})$, and $(3, \frac{\pi}{2})$, compute $DT(r_0, \theta_0)$.

2. The basis $\{\bar{e}_1, \frac{\pi}{4}\bar{e}_2\}$ has been drawn at each of the points $(1, \frac{\pi}{2}), (2, \frac{\pi}{2}), (3, \frac{\pi}{2})$. For each of these points (r_0, θ_0), compute
$$DT(r_0, \theta_0)(\bar{e}_1) \quad \text{and} \quad DT(r_0, \theta_0)(\tfrac{\pi}{4}\bar{e}_2).$$

3. Plot your answers to Question 2 on the picture of the range R above.

4. In what sense is $DT(r_0, \theta_0)$ a linear approximation of T near (r_0, θ_0)?