Multivariable Functions (and surfaces in $\mathbb{R}^3$)

Notation:

$$f: X \longrightarrow Y$$

1-1
onto

X: domain

Y: codomain

Ex $f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \sin(x^3 + y)$$

The **domain** is set of possible inputs to f.

The **range** is the set actual outputs of f.

↑ always a subset of codomain.

In the example above:

domain $= \mathbb{R}^2$

range $= [-1, 1]$.

A function $f: \mathbb{R}^n \to \mathbb{R}$ (*) is called a

scalar-valued function.

A function $f: \mathbb{R}^n \to \mathbb{R}^m$ ($m > 1$) is called a

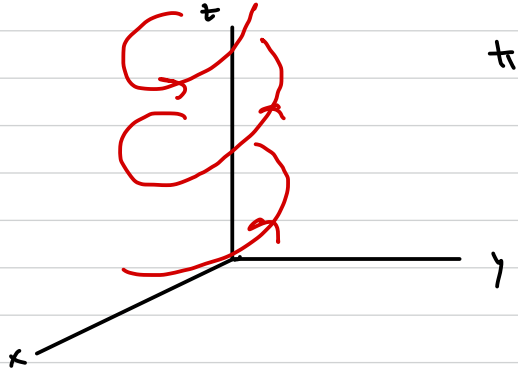
vector-valued function.

e.g. $f: \mathbb{R}^2 \to \mathbb{R}^3$

$$f(x,y) = \left(\cos(xy),\ |x+y^2|,\ \frac{x}{y^4}\right)$$

these are called component functions.

Note: $f: \mathbb{R} \to \mathbb{R}^3$ (*) is a curve -- swept out by tips of vectors.



Ex: vector eqn of line.