

For now: focus on scalar-valued functions.

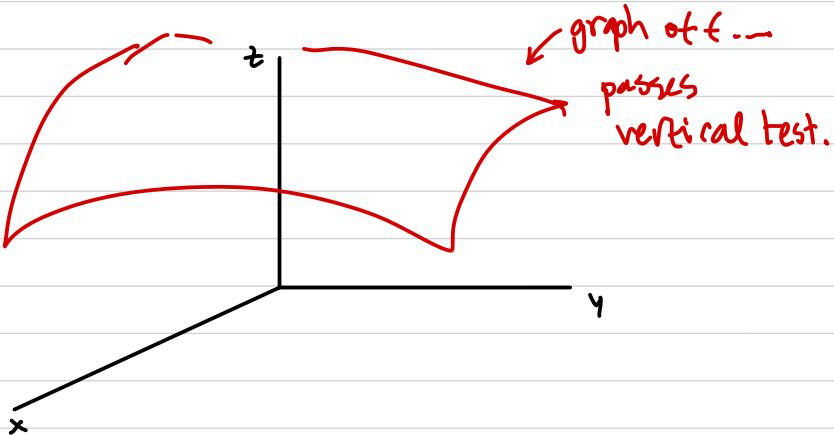
Recall: graphing $f: \mathbb{R} \rightarrow \mathbb{R} \rightsquigarrow$ we set $y = f(x)$

↪ similarly, for $f: \mathbb{R}^2 \rightarrow \mathbb{R} \rightsquigarrow$ we set $z = f(x,y)$

So, for $f: \mathbb{R}^2 \rightarrow \mathbb{R}$, the graph of f is the set

$$\{(x,y, f(x,y)) \mid (x,y) \in \text{domain of } f\} \subset \mathbb{R}^3$$

and we depict it as:



More generally, if $f: \mathbb{R}^m \rightarrow \mathbb{R}$ the graph of f is:

$$\left\{ (\bar{x}, f(\bar{x})) \mid \bar{x} \in \text{domain of } f \right\} \subset \mathbb{R}^{m+1}.$$

$\begin{matrix} \uparrow & \uparrow \\ \text{in } \mathbb{R}^m & \text{in } \mathbb{R} \end{matrix}$

Q: How to draw graph $f: \mathbb{R}^2 \rightarrow \mathbb{R}$?

↪ tool: level/contour curves

* (these have theoretical value beyond graphing.) *