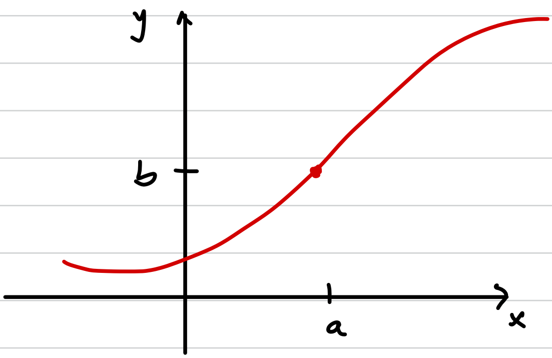
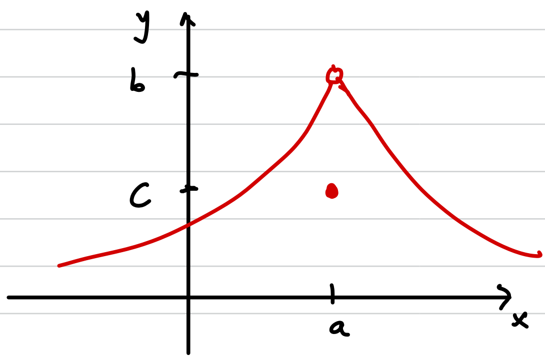
Limits and Continuity

Recall : single variable limits :

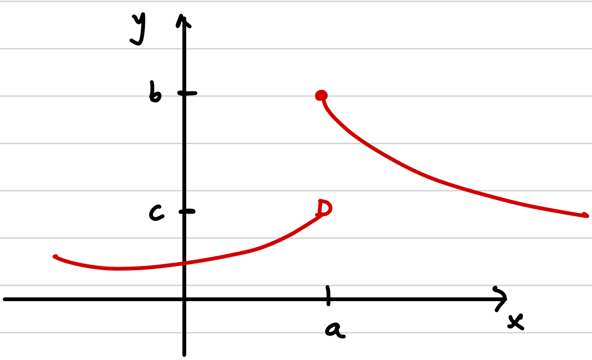


$\lim_{x \to a} f(x) = b$



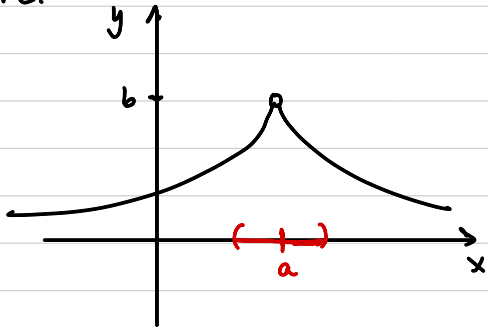
$\lim_{x \to a} f(x) = b$

(though $f(a) = c$)



$\lim_{x \to a} f(x)$ does not exist.

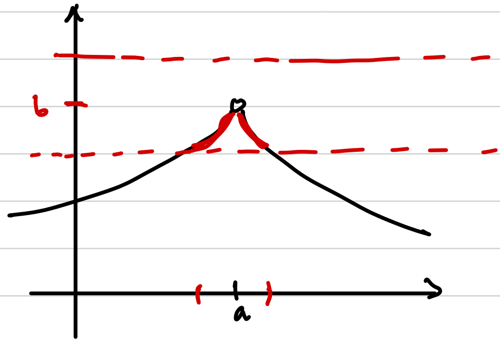
Idea: If x is close to a, i.e.



$|x-a|$ is small

single variable distance from x to a

then $f(x)$ is close to b, i.e.



$|f(x)-b|$ is small.

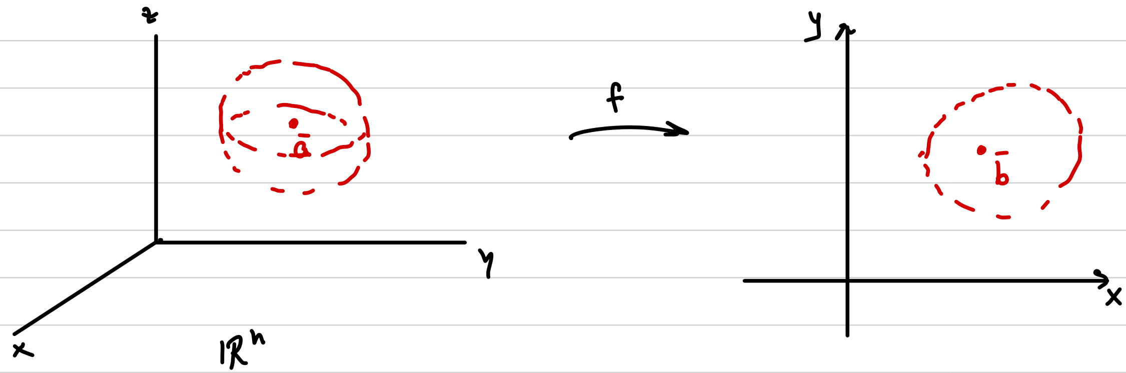
single variable distance from $f(x)$ to b.

This can be made precise using ε-δ definition of limit.

(Math 323 - Real Analysis)

Multivariable limit is based on same idea:

Sps $f: \mathbb{R}^n \to \mathbb{R}^m$ (e.g. $f: \mathbb{R}^3 \to \mathbb{R}^2$)



Say $\lim_{\bar{x} \to \bar{a}} f(\bar{x}) = \bar{b}$ if...

vector in codomain.

vector in domain

whenever $\bar{x}$ is close to $\bar{a}$ (i.e. $|\bar{x} - \bar{a}|$ is small)

distance in $\mathbb{R}^n$

then $f(\bar{x})$ is close to $\bar{b}$ (i.e. $|f(\bar{x}) - \bar{b}|$ is small)

distance in $\mathbb{R}^m$

Can turn this into a rigorous $\varepsilon - \delta$ defn of the limit.

epsilon delta