Official defn of limit = hard to compute with.

Tools:

where denom $\neq 0$

1. Limits behave well with $+$, $-$, $\times$, $\div$, scalar mult.

e.g. $$\lim_{\vec{x} \to \bar{a}} f(\bar{x}) + g(\bar{x}) = \lim_{\bar{x} \to \bar{a}} f(\bar{x}) + \lim_{\bar{x} \to \bar{a}} g(\bar{x})$$

(polynomials)

2. For vector-valued functions, can compute limits componentwise.

$F: \mathbb{R}^n \to \mathbb{R}^m$

$$F = (f_1, f_2, \ldots, f_m)$$

scalar component functions

can compute these limits individually to find limit of F.

3. Continuity:

Defn Say $F: \mathbb{R}^n \to \mathbb{R}^m$ is continuous at $\bar{a}$ if

$$\lim_{\bar{x} \to \bar{a}} f(\bar{x}) = f(\bar{a}).$$

so limit exists, f is defined at $\bar{a}$, and they are equal.

So: if f cts at $\bar{a}$, can compute limit by:

- first arguing that the function is cts at $\bar{a}$
- then plug $\bar{a}$ into f to find limit.

Further: sin, cos, ln, $e^{( )}$, $( )^n$, $\sqrt{\ }$, $+$, $-$, $\times$, $\div$

are all cts on their domains

as are composites.

Ex $$\lim_{(x,y) \to (1,1)} \left(\frac{\cos(x^2+y^2)}{\sqrt{xy}},\ x+y^2 \right)$$

Note: scalar component functions are cts at $(1,1)$ b/c they composites of functions cts at $(1,1)$.

So: find limit by plugging in: value is: $\left(\frac{\cos 2}{1}, 2 \right)$.