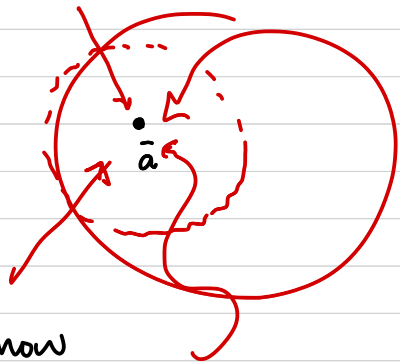
Note: In order for a limit $f:\mathbb{R}^n \to \mathbb{R}^m$ to exist and equal $\bar{b}$ at $\bar{a}$, no matter which path you use to approach $\bar{a}$, the function values along the path must approach $\bar{b}$.



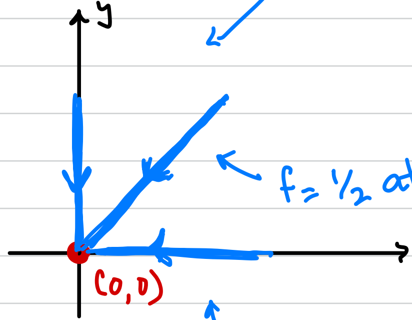
can use to show that a limit doesn't exist.

Ex $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$?

along $x=0$?

nicely defined if $y \neq 0$ ... equal to 0 for all $y \neq 0$.

If $x=0$: $\displaystyle\lim_{y\to 0} \frac{0}{y^2} = 0.$



$f = 1/2$ at every pt on this path

$f = 0$ at every pt on this path

along $y=0$?

If $y=0$, $\displaystyle\lim_{x\to 0} \frac{0}{x^2} = 0.$

not equal, so limit d.n.e.

along $x=y$?

$$\lim_{x\to 0} \frac{xx}{x^2+x^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$$

In polar coordinates:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$\lim_{r\to 0} \frac{r\cos\theta \, r\sin\theta}{r^2} = \lim_{r\to 0} \cos\theta \sin\theta$$

this function gives diff values for diff (fixed) choices of θ ... so limit d.n.e.