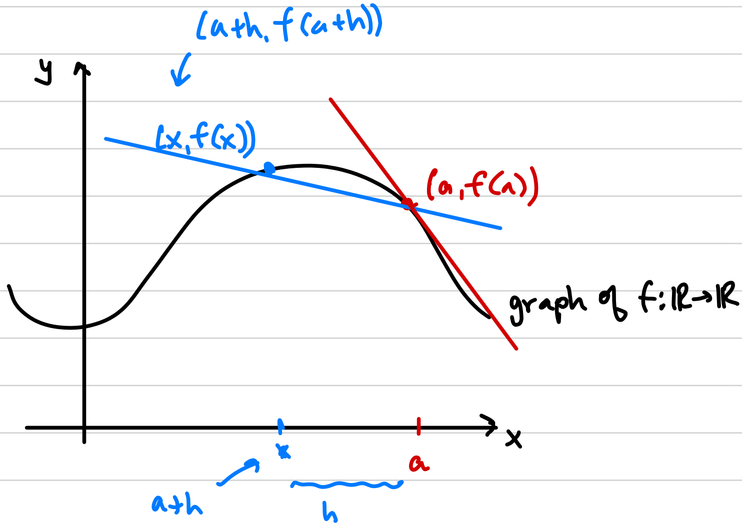
Partial Derivatives and The Total Derivative

Recall:



$$\left.\frac{df}{dx}\right|_{x=a} = f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

Defn Sps $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y)$

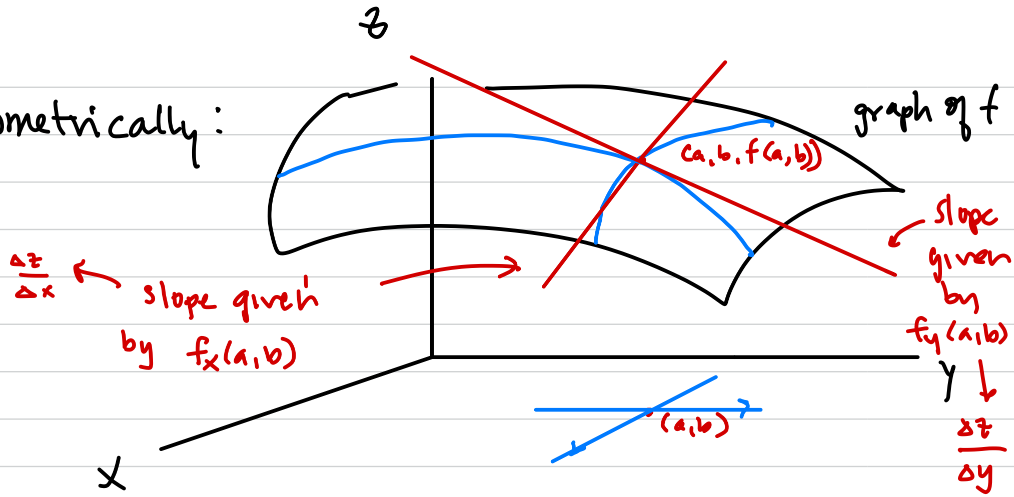
The partial derivative of f with respect to x is:

$$\underbrace{\frac{\partial f}{\partial x} = f_x = D_x f}_{\text{notations.}} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

this would be deriv at (x,y)

Notes: 1. f_y computed similarly. 2. Defn extends naturally to $f: \mathbb{R}^n \to \mathbb{R}$.

Geometrically:



Algebraically:

__Ex__ Sps. $f(x,y,z) = x\cos(xyz)$. Find $\frac{\partial f}{\partial z}$ at $(1, 2, \frac{\pi}{4})$.

Idea: $\frac{\partial f}{\partial z}$ measures how f varies as z varies.

→ consider x and y as constants.

$$\left.\frac{\partial f}{\partial z}\right|_{(x,y,z)} = -x\sin(xyz)\cdot xy = -x^2 y\sin(xyz)$$

$$\left.\frac{\partial f}{\partial z}\right|_{(1,2,\pi/4)} = -\sin\left(\frac{\pi}{2}\right)(2) = -2.$$