We can compute higher-order partials:

e.g. $f(x,y,z) = x\cos(xyz)$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial z}$

$$\frac{\partial^2 f}{\partial x \partial z} \quad \text{means:} \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right)$$

same.

Alternate: f_{zx} means: $(f_z)_x$

<u>Ex</u> $f(x,y,z) = x\cos(xyz)$

$$\frac{\partial f}{\partial z} = -x^2 y \sin(xyz)$$

$$\frac{\partial^2 f}{\partial x \partial z} = -2xy\sin(xyz) - x^2 y\cos(xyz)(yz)$$

$$= -2xy\sin(xyz) - x^2y^2z\cos(xyz).$$

Useful Thm: Sps $f: \mathbb{R}^n \to \mathbb{R}$ is such that

f_{x_i} and $f_{x_i x_j}$ are cts for all i, j.

Then

$$f_{x_i x_j} = f_{x_j x_i} \qquad \text{for all } i, j.$$

"equality of mixed partials"

Ex $f(x, y, z) = x \cos(xyz)$

$$\frac{\partial f}{\partial x} = \cos(xyz) - xyz \sin(xyz)$$

$$\frac{\partial^2 f}{\partial z \partial x} = -\sin(xyz)(xy) - xy\sin(xyz) - xyz\cos(xyz)(xy)$$

$$= -2xy\sin(xyz) - x^2y^2z\cos(xyz).$$

$$= \frac{\partial^2 f}{\partial x \partial z}. \quad ✓$$