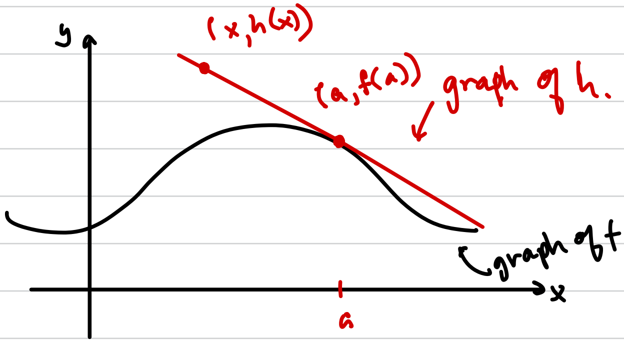
Tangent Planes and Differentiability

Consider:

$f: \mathbb{R} \to \mathbb{R}$



Let $h(x) = mx + b$ be eqn of tangent line to graph of f at $x = a$.

$$\frac{\Delta y}{\Delta x} = \frac{h(x) - f(a)}{x - a} = f'(a)$$

aside:

$f(a) + f'(a)(x-a)$

$+ \frac{f''(a)}{2!}(x-a)^2$

$$\Rightarrow h(x) = f(a) + f'(a)(x-a)$$

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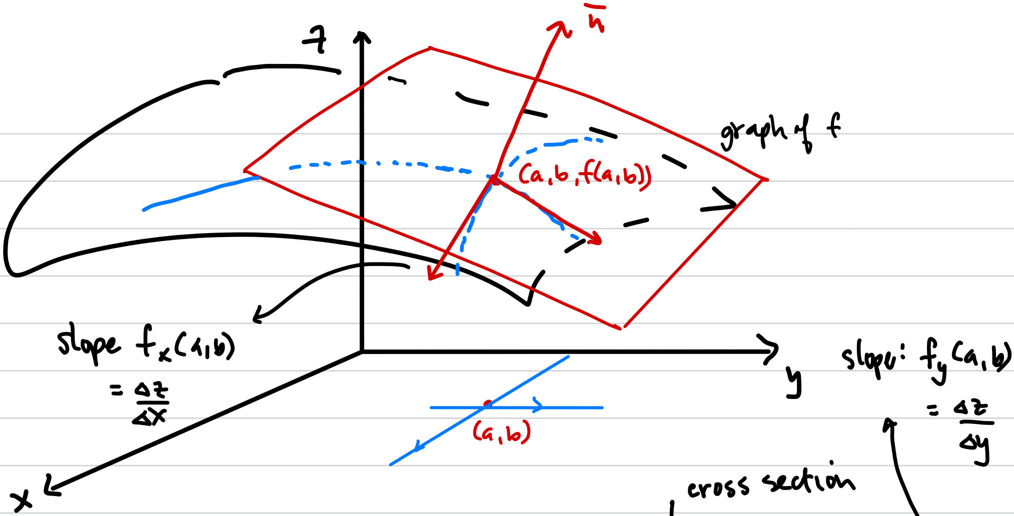
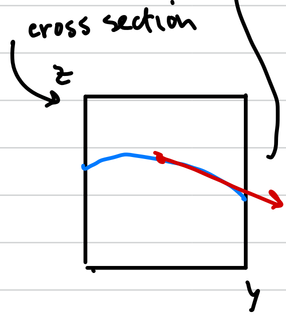
Now, sps $f: \mathbb{R}^2 \to \mathbb{R}$.

WANT: function $h(x,y)$ that gives tangent plane to graph of $f(x,y)$ at (a,b) if it exists.



GOAL: tangent plane.



point on plane: $(a, b, f(a,b))$

2 vectors $\underline{\underline{\text{in}}}$ plane: $(0, 1, f_y(a,b))$

$(1, 0, f_x(a,b))$

normal is cross product: $(f_x(a,b), f_y(a,b), -1)$

$$\text{Plane:} \quad 0 = \bar{n} \cdot (\underbrace{\vec{r}}_{(x,y,h(x,y))} - \underbrace{\bar{r}_0}_{(a,b,f(a,b))})$$

$$= f_x(a,b)(x-a) + f_y(a,b)(y-b) - 1(h(x,y) - f(a,b))$$

$$\Rightarrow \underline{h(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

(compare: tan line: $h(x) = f(a) + f'(a)(x-a)$.)