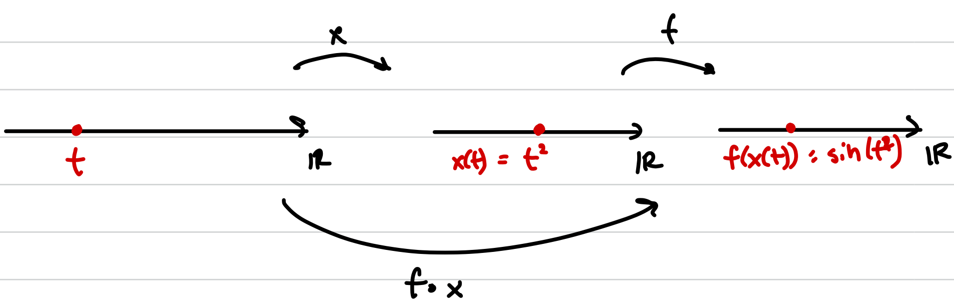
The Chain Rule

↳ a rule for differentiating composite functions.

<u>Ex</u> $f : \mathbb{R} \to \mathbb{R}$

$$f(t) = \underbrace{\sin}_{f(x) = \sin x}(\underbrace{t^2}_{x(t) = t^2})$$

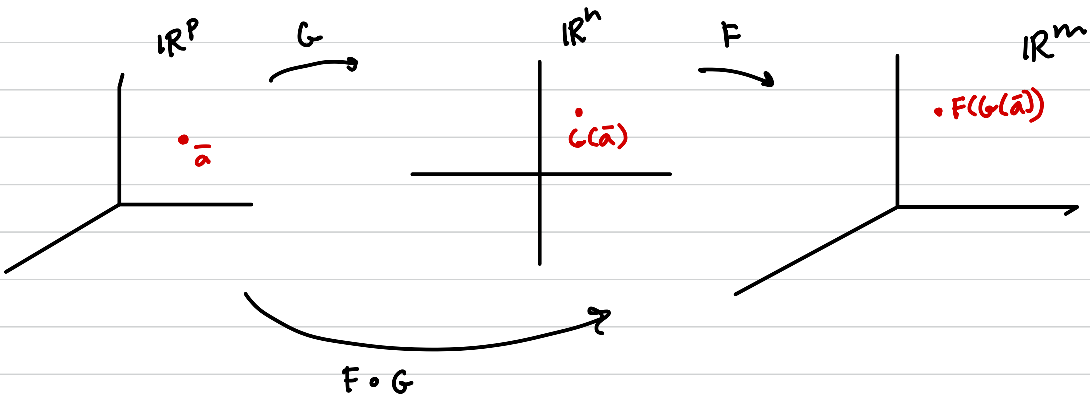
$$\frac{df}{dt} = \cos(t^2)(2t) = f'(x(t))\,x'(t)$$



Now, the multivariable version:

Thm (Chain Rule)

Sps. $F: \mathbb{R}^n \to \mathbb{R}^m$ and $G: \mathbb{R}^p \to \mathbb{R}^n$ are such that the range of G is contained in the domain of F.



Sps G is diff'ble at $\bar{a}$ and F is diff'ble at $G(\bar{a})$.

Then $F \circ G$ is diff'ble at $\bar{a}$ and

$$D(F\circ G)_{\bar{a}} : \mathbb{R}^p \to \mathbb{R}^m \text{ and}$$

$$D(F\circ G)_{\bar{a}} = DF(G(\bar{a}))\,DG(\bar{a})$$

composite of L.T.'s (?!)

matrix mult.