$\underline{\text{Ex}}$ $\quad G(s,t) = (\overset{x}{st^2}, s-t)$ $\qquad F(x,y) = (x^2+y, xy)$

① Direct substitution:

$$F\circ G(s,t) = (\overset{x^2+y}{s^2t^4+s-t}, \overset{xy}{s^2t^2-st^3})$$

$$D(F\circ G)_{(s,t)} = \begin{array}{cc} \frac{\partial(F\circ G)}{\partial s} & \frac{\partial(F\circ G)}{\partial t} \\ \multicolumn{2}{c}{\begin{bmatrix} 2st^4+1 & 4s^2t^3-1 \\ 2st^2-t^3 & 2s^2t-3st^2 \end{bmatrix}} \end{array} \quad \text{A}$$

② Chain rule:

$$DF(x,y) = \begin{bmatrix} 2x & 1 \\ y & x \end{bmatrix} \qquad DF(G(s,t)) = \begin{bmatrix} 2st^2 & 1 \\ s-t & st^2 \end{bmatrix} \quad \text{B}$$

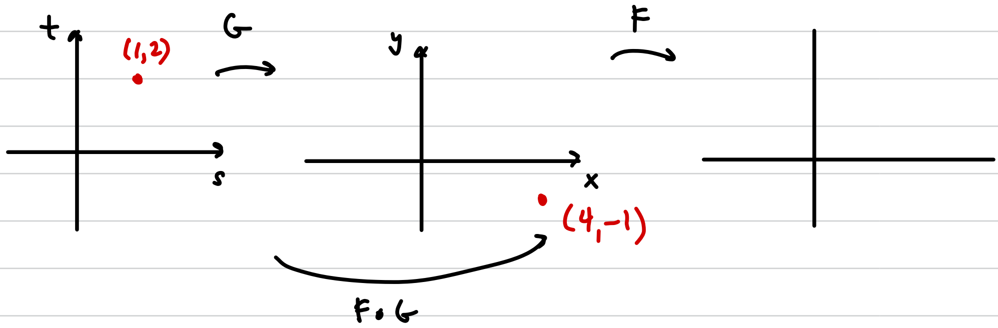
$$DG(s,t) = \begin{bmatrix} t^2 & 2st \\ 1 & -1 \end{bmatrix} \quad \text{C}$$

Chain rule says:

A = B C ✓

$G(s,t) = (st^2, s-t) \qquad F(x,y) = (x^2+y, xy)$

At $(s,t) = (1,2)$, $\quad G(1,2) = (4,-1)$:



$$D(F\circ G)(1,2) = DF(4,-1)\, DG(1,2)$$

$$\begin{bmatrix} 33 & 31 \\ 0 & -8 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 1 & -1 \end{bmatrix}$$