Another way to see the chain rule, for scalar-valued functions (or component functions):

<u>Ex</u>: Sps $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z)$

Ex: $f(x,y) = xe^y$

$x = \cos t \quad y = \ln t$

Sps. $\bar{x} = (x,y,z)$ depends on s and t.

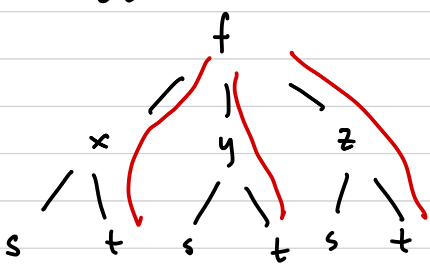
So, we can consider $(f \circ \bar{x})(s,t)$.

$D(f\circ\bar{x})(s,t)$ $\quad 1\times 2$ *

$$\begin{bmatrix} \frac{\partial f}{\partial s} & \frac{\partial f}{\partial t} \end{bmatrix} =$$

$$\underset{1\times 3}{\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} D\bar{x} \end{bmatrix}}$$

Q: What is $\frac{\partial f}{\partial t}$?

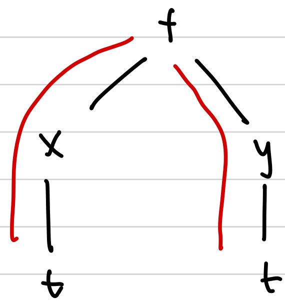


$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t}$$

* Tree diagram gives same result as matrix multiplication.

<u>Ex</u> $f(x,y) = xe^y$ $\quad x = \cos t \quad y = \ln t$

What is $\dfrac{df}{dt}$?



$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= e^y(-\sin t) + xe^y\left(\frac{1}{t}\right)$$ ⟵ get everything in terms of t

$$= e^{\ln t}(-\sin t) + \cos t \, e^{\ln t}\left(\frac{1}{t}\right)$$

$$= -t\sin t + \frac{t\cos t}{t}$$

$$= -t\sin t + \cos t. \checkmark$$