How to compute?

Sps $f: \mathbb{R}^n \to \mathbb{R}$ is differentiable.

Define $\nabla f(\bar{a})$, a vector, by

$$\underbrace{\nabla f(\bar{a})}_{\text{"gradient of } f\text{"}} = \left(\frac{\partial f}{\partial x_1}(\bar{a}), \frac{\partial f}{\partial x_2}(\bar{a}), \dots, \frac{\partial f}{\partial x_n}(\bar{a}) \right)$$

Note! same entries at $DF(\bar{a}) = [\quad]$

Note: gradient vector is helpful for computing directional derivatives, but contains other interesting information as well.

Now consider:

$\bar{v}$ = unit vector

Calc. I

If $\bar{x}(t) = \bar{a} + t\bar{v}$, note $f \circ \bar{x} : \mathbb{R} \to \mathbb{R}$.

We get:

$$D_{\bar{v}} f(\bar{a}) = \lim_{t \to 0} \frac{f(\bar{a} + t\bar{v}) - f(\bar{a})}{t}$$

$\bar{x}(0) = \bar{a}$

$$= \lim_{t \to 0} \frac{(f \circ \bar{x})(t) - (f \circ \bar{x})(0)}{t}$$

calc I deriv.

$$= \frac{d}{dt}\Big|_{t=0} (f \circ \bar{x})$$

$\bar{x}(t) = \bar{a} + t\bar{v}$

$(a_1 + tv_1, a_2 + tv_2, \dots, a_n + tv_n)$

change of notation.... to total derivative

$$= D(f \circ \bar{x})(0)$$

$\bar{x}(0) = \bar{a}$

$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

chain rule

$$= Df(\bar{x}(0))\, D\bar{x}(0)$$

$$= Df(\bar{a}) \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial f}{\partial x_1}(\bar{a}) & \frac{\partial f}{\partial x_2}(\bar{a}) & \cdots & \frac{\partial f}{\partial x_n}(\bar{a}) \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ \vdots \\ v_n \end{bmatrix}$$

reexpress in vector notation

$$= \nabla f(\bar{a}) \cdot \bar{v}$$

So: to compute: $\boxed{D_{\bar{v}} f(\bar{a}) = \nabla f(\bar{a}) \cdot \bar{v}}$

Ex $D_{\bar{u}}(3xy^2)$ in direction of $(2,3)$? At the pt. $(1,1)$?

First: normalize $(2,3)$: $\dfrac{1}{\sqrt{4+9}}(2,3) = \underbrace{\left(\dfrac{2}{\sqrt{13}}, \dfrac{3}{\sqrt{13}}\right)}_{\bar{u}}$

$$\nabla f(x,y) = (3y^2, 6xy)$$

$$\nabla f(1,1) = (3,6)$$

$\Rightarrow$ $D_{\bar{u}}(3xy^2)$ at $(1,1)$ is:

$$(3,6)\cdot\left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right) = \frac{24}{\sqrt{13}}.$$