## Remarks

1. In general, for $f(x,y)$, think of graph $z = f(x,y)$ as 0-level surface of

$$F(x,y,z) = f(x,y) - z.$$

see earlier notes!

Then $\nabla F(a,b,f(a,b)) = (f_x(a,b),\ f_y(a,b),\ -1)$

So tangent plane at $(a,b,f(a,b))$ given by:

$$\nabla F(a,b,f(a,b)) \cdot (x-a,\ y-b,\ z-f(a,b)) = 0$$

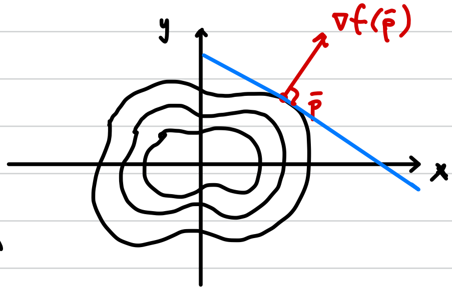
$$\Rightarrow (f_x(a,b),\ f_y(a,b),\ -1) \cdot (x-a,\ y-b,\ z-f(a,b)) = 0$$

$$\Rightarrow \quad z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

compare with $h(x,y)$ from before. This is a special case of more general principle.

2. Same argument shows that for $f: \mathbb{R}^2 \to \mathbb{R}$,

$\nabla f$ is orthogonal to level curves.




domain of $f$

recall: $\nabla f$ points in direction of steepest ascent.