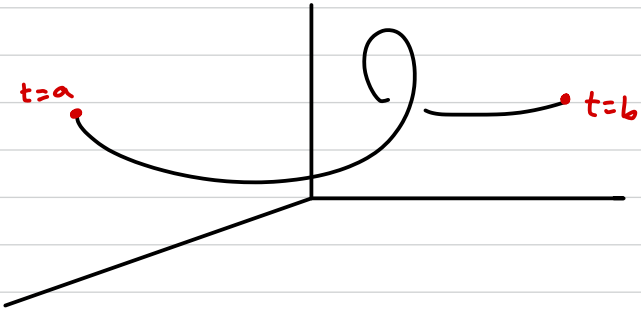
Arclength

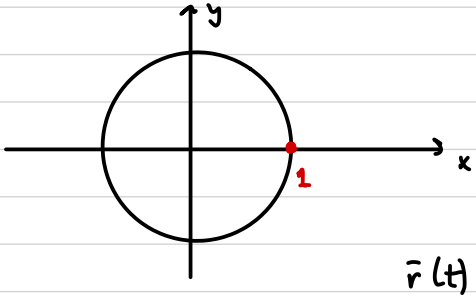
Given a parametrized curve,

$$\bar{r}(t) : \mathbb{R} \to \mathbb{R}^n$$

for $a \leq t \leq b$, we can calculate the arclength of the segment



E.g. Arclength of circle, radius 1 is : 2π ← circumference



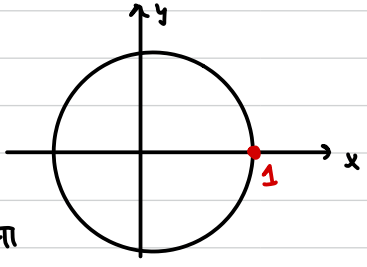
$\bar{r}(t) = (\cos t, \sin t)$

$0 \leq t \leq 2\pi \ldots$

Defn The length of a curve C parametrized by $\bar{r}: \mathbb{R} \to \mathbb{R}^n$ between $t=a$ and $t=b$ is:

$$L(C) = \int_a^b |\bar{r}'(t)|\, dt$$

Ex circle, radius 1.



$\bar{r}(t) = (\cos t, \sin t) \quad 0 \le t \le 2\pi$

$\bar{r}'(t) = (-\sin t, \cos t) \rightsquigarrow |\bar{r}'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2}$

$= 1$

So $L(\text{circle}) = \int_0^{2\pi} 1\, dt = t\Big|_0^{2\pi} = \boxed{2\pi}$.