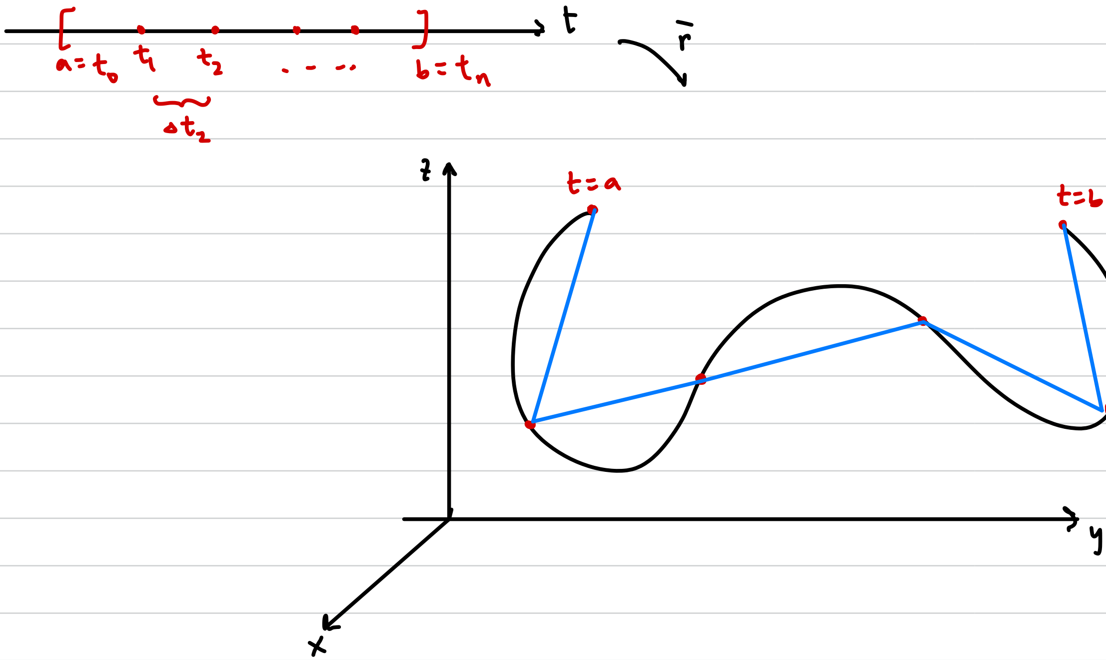
Why?

↳ i.e. why does $\int_a^b |\bar{r}'(t)|\,dt$ give length of C?

(sketch of proof)



- Divide $[a,b]$ into n subintervals, width Δt_i
- Consider line segment b/w $\bar{r}(t_{i-1})$ and $\bar{r}(t_i)$.

↳ length $= \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2 + (\Delta z_i)^2}$

lengths of segments

$$\text{Length of curve} \approx \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2}$$

$$\|$$

as $\Delta t_i \to 0$,

$n \to \infty$

take limit as $\Delta t_i \to 0$

calculus

$$\lim_{\Delta t_i \to 0} \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2} \cdot \frac{\Delta t_i}{\Delta t_i}$$

$\frac{\Delta x_i}{\Delta t_i} \rightsquigarrow \frac{dx}{dt}$

$$= \lim_{\Delta t_i \to 0} \sum_{i=1}^{n} \sqrt{\left(\frac{\Delta x_i}{\Delta t_i}\right)^2 + \left(\frac{\Delta y_i}{\Delta t_i}\right)^2 + \left(\frac{\Delta z_i}{\Delta t_i}\right)^2} \, \Delta t_i$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt$$

$\bar{r}(t) = (x(t), y(t), z(t))$

$$= \int_a^b |\bar{r}'(t)| \, dt.$$

Tada!