Ex. Length of curve parametrized by

$$\bar{r}(t) = \left(t^2, \frac{2}{3}(2t+1)^{3/2}\right), \quad b/w \quad 0 \le t \le 4.$$

$$\bar{r}'(t) = \left(2t, (2t+1)^{1/2}(2)\right)$$

$$L(C) = \int_0^4 \sqrt{(2t)^2 + (2(2t+1)^{1/2})^2}\, dt$$

$$= \int_0^4 \sqrt{4t^2 + 8t + 4}\, dt$$

$$= 2\int_0^4 \sqrt{t^2 + 2t + 1}\, dt$$

$$= 2\int_0^4 \sqrt{(t+1)^2}\, dt$$

$$= 2\int_0^4 (t+1)\, dt$$

$$= 2\left(\frac{t^2}{2} + t\right)\Big|_0^4$$

$$= t^2 + 2t \Big|_0^4$$

$$= 16 + 8 - 0 = \boxed{24}$$

Important observation:

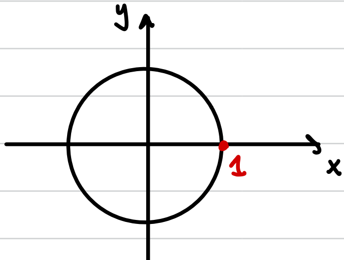
The formula for the length of a curve segment is based on a choice of parametrization.

$L(C) = \int_a^b |\bar{r}'(t)|\,dt$

But for any curve, there are many ways to parametrize it.

Q: do all choices of parametrizations of C produce the same value for $L(C)$?

A: YES. Length indep. of choice of parametrization.



If $\bar{r}'(t) \neq 0$ for all t, can reparametrize by arc length... steady, unit-speed parametrization.