Vector Fields

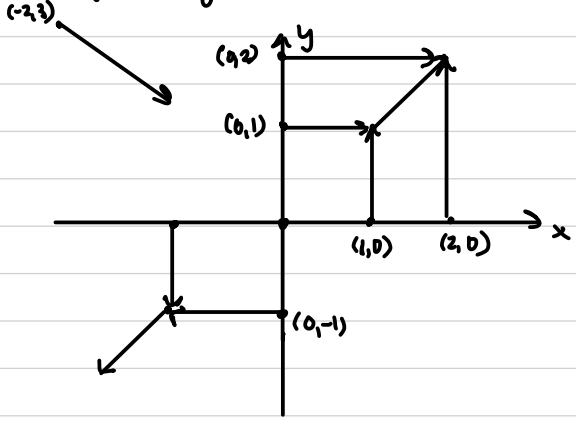
A vector field is a function

same

$F: \mathbb{R}^n \to \mathbb{R}^n$

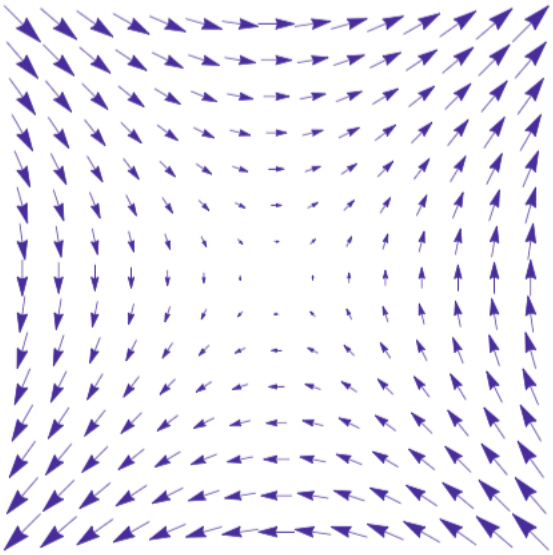
Idea: at every pt in $\mathbb{R}^n$, place an n-vector.

Ex $F(x,y) = (y, x)$



Example

$F(x, y) = (y, x)$



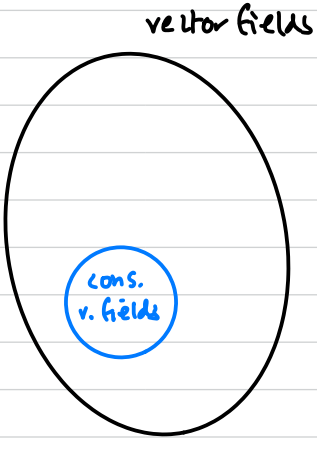
Important Example

Sps $f: \mathbb{R}^n \to \mathbb{R}$ is a scalar function.*

The gradient of f,

$$\nabla f(\bar{x}) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

is a vector field.



Ex $f(x,y) = xy$ ← previous example.

$\nabla f(x,y) = (y, x)$

Note: not every vector field is the gradient of some scalar function f.

← special property.

If $\bar{F} = \nabla f$ for some f, $\bar{F}$ is called conservative and f is called a potential function of $\bar{F}$.