Operations on Vector Fields

gradient

grad : scalar functions $\longrightarrow$ vector fields

divergence

div : vector fields $\longrightarrow$ scalar functions

curl : vector fields in $\mathbb{R}^3$ $\longrightarrow$ vector field in $\mathbb{R}^3$.

We've already talked about the gradient...

Divergence

Defn Sps $\bar{F}: \mathbb{R}^n \to \mathbb{R}^n$ so

$$\bar{F}(\bar{x}) = (F_1(\bar{x}), F_2(\bar{x}), \ldots, F_n(\bar{x}))$$

component functions.

(e.g. $F(\bar{x}) = (xyz, e^{xy}, z^2 \cos x)$)

Then

$\operatorname{div} F(\bar{x}) : \mathbb{R}^n \to \mathbb{R}$ * is given by

$$\operatorname{div} F(\bar{x}) = \underbrace{\frac{\partial F_1}{\partial x_1}(\bar{x}) + \frac{\partial F_2}{\partial x_2}(\bar{x}) + \dots + \frac{\partial F_n}{\partial x_n}(\bar{x})}_{\text{a scalar-valued function}}$$

Ex $F(\bar{x}) = (xyz,\ e^{xy},\ z^2 \cos x)$

$$\operatorname{div} F(\bar{x}) = yz + xe^{xy} + 2z\cos x.$$

Note on notation: If we let the symbol ∇ "del" represent the operation

$$\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_n} \right)$$

a "vector" of operators

scalar-valued function

then the gradient of f can be denoted ∇f

$\left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$

vector-valued function

and the divergence of $\bar{F}$ can be denoted $\nabla \cdot \bar{F}$.

shorthand / alternate, indicative notation.

Ex $\bar{F}(x,y,z) = (xyz, e^{xy}, z^2\cos x)$

symbolic "dot product"

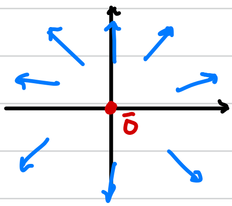
$$\operatorname{div}\bar{F} = \nabla \cdot \bar{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (xyz, e^{xy}, z^2\cos x)$$

simply an alternate notation ... indicative of the process of computing $\operatorname{div}\bar{F}$

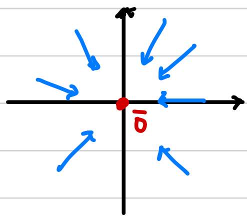
$$= \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(e^{xy}) + \frac{\partial}{\partial z}(z^2\cos x)$$

$$= yz + xe^{xy} + 2z\cos x.$$

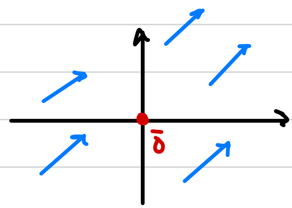
Given a vector field $\bar{F}$, suppose $\bar{F}$ describes fluid flow. Then $\operatorname{div}\bar{F}(\bar{a})$ measures tendency of fluid to flow away from (or toward) $\bar{a}$.



$\operatorname{div} F(\bar{a})$ positive



$\operatorname{div} F(\bar{a})$ negative



$\operatorname{div} F(\bar{a}) = 0.$