Curl Sps $\bar{F}: \mathbb{R}^3 \to \mathbb{R}^3$ is a 3-dim'l vector field.

$$\bar{F}(x,y,z) = (F_1(x,y,z), F_2(x,y,z), F_3(x,y,z))$$

Then curl $\bar{F}$ is the 3-dim'l vector field

$$\text{curl } \bar{F} = \text{"det"} \begin{array}{cc} \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{bmatrix} & \begin{array}{l} \\ \leftarrow \nabla \\ \leftarrow \bar{F} \end{array} \end{array}$$

Note on notation: curl $\bar{F}$ can also be denoted $\nabla \times \bar{F}$

alternate, indicative notation.

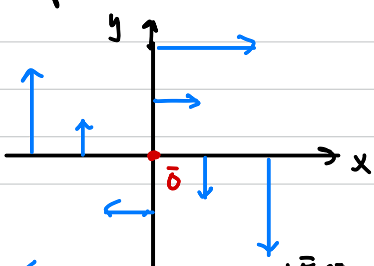
Ex $\bar{F} = (xyz,\ \sin(xy),\ x+2y-z)$

$$\operatorname{curl} \bar{F} = \text{"det"} \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & \sin(xy) & x+2y-z \end{bmatrix}$$

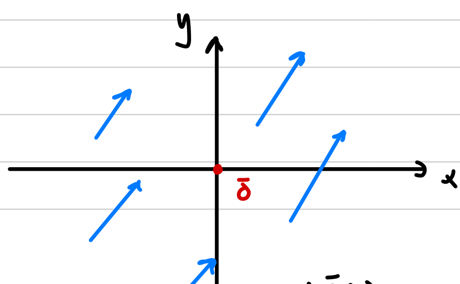
$$= (2-0)\bar{i} - (1 - xy)\bar{j} + (y\cos(xy) - xz)\bar{k}$$

$$= (2,\ xy-1,\ y\cos(xy) - xz).$$

Given a vector field $\bar{F}$, curl $\bar{F}$ measures the tendency of $\bar{F}$ to "swirl".



curl $\bar{F}(\bar{0})$ nonzero



curl $\bar{F}(\bar{0})$ zero

Thm Sps $f : \mathbb{R}^3 \to \mathbb{R}$ has cts. 1^{st} and 2^{nd} order partials.

Then

$$\text{curl } \nabla f = \bar{0}$$ ← vector field

grad

curl

div

Δ

Laplacian

proof

$$\text{grad } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\text{curl}(\text{grad } f) = \text{"det"} \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix}$$ ← grad f

$$= \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}, \frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}, \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right)$$

$= \bar{0}$ b/c equality of mixed partials.

Thm Sps $\bar{F} : \mathbb{R}^3 \to \mathbb{R}^3$ has component functions with cts 1^{st} and 2^{nd} order partials. Then

$$\text{div}(\text{curl } \bar{F}) = 0$$ ← scalar valued function

proof: exercise!