Ex If $f(x,y) = e^{xy} + x^2 + y$, find $p_2(\bar{x})$ at $(0,0)$.

$$p_2(\bar{x}) = f(\bar{0}) + Df(\bar{0})(\bar{x}-\bar{0}) + \frac{1}{2}(\bar{x}-\bar{0})^T Hf(\bar{0})(\bar{x}-\bar{0})$$

$f(0,0) = 1$

$Df(x,y) = [\, ye^{xy} + 2x \quad xe^{xy} + 1 \,]$

$Df(0,0) = [\, 0 \quad 1 \,]$

$$Hf(x,y) = \begin{bmatrix} y^2e^{xy} + 2 & e^{xy} + xye^{xy} \\ e^{xy} + xye^{xy} & x^2e^{xy} \end{bmatrix}$$

$$Hf(0,0) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \qquad \bar{x} - \bar{0} = \begin{bmatrix} x-0 \\ y-0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

So:

$$p_2(\bar{x}) = 1 + [\, 0 \quad 1 \,]\begin{bmatrix} x \\ y \end{bmatrix} + \frac{1}{2}[\, x \quad y \,]\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

If you multiply it out:

$$\frac{1}{2}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2x+y \\ x \end{bmatrix}$$

$$= \frac{1}{2}(2x^2 + xy + xy)$$

$$= x^2 + xy$$

So $p_2(\bar{x}) = 1 + y + x^2 + xy$

an even better approx. of $f(x,y) = e^{xy} + x^2 + y$ near $(0,0)$.