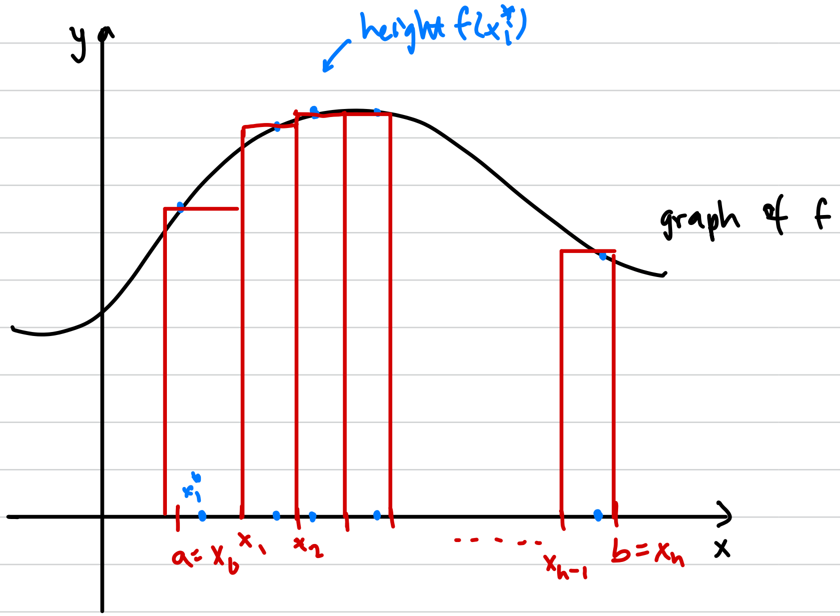
Double Integrals

Recall:



To define $\int_a^b f(x)\,dx$:

- divide $[a,b]$ into n subintervals, length $\Delta x_i = x_i - x_{i-1}$
- for each i, choose $x_i^* \in [x_{i-1}, x_i]$.
- calculate $f(x_i^*)\Delta x_i$ area one rectangle
- Then: $\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$ Riemann sum

* *Calculus!

An Interpretation:

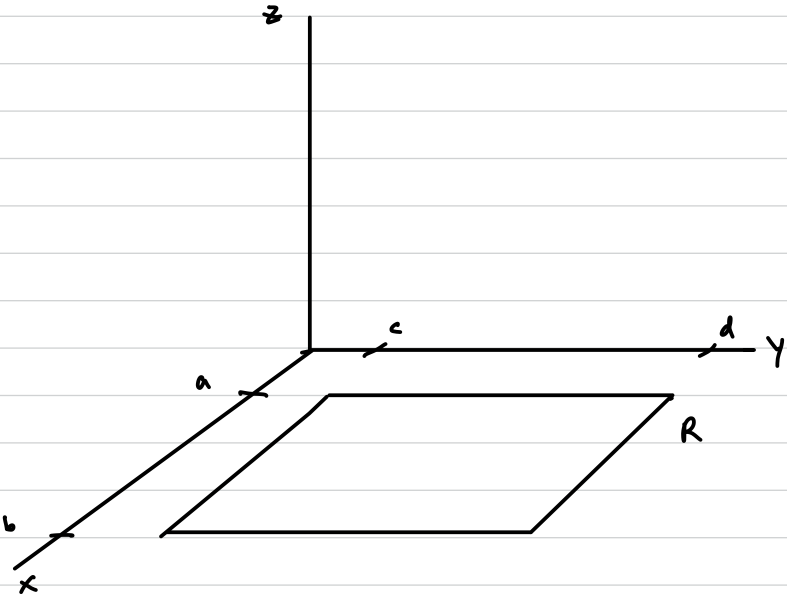
area below x-axis counts neg.

$\int_a^b f(x)\,dx$ = area b/w graph of f and x-axis.

*

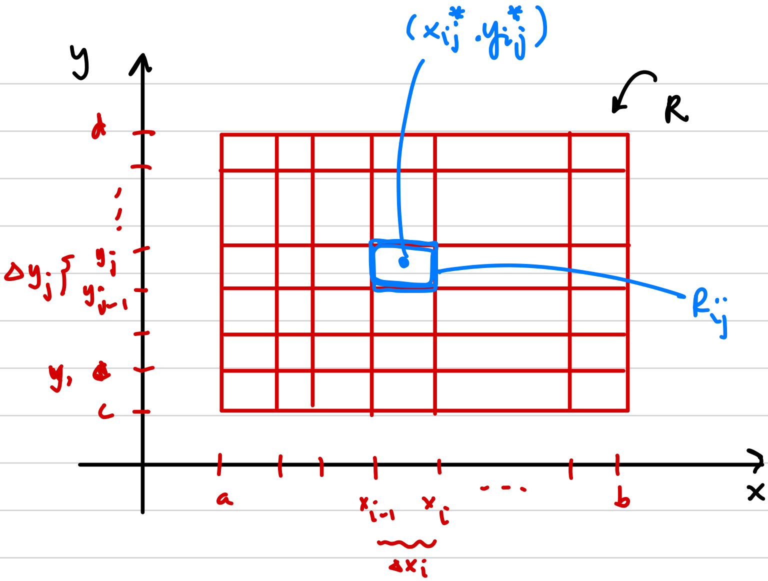
Now suppose $f : \mathbb{R}^2 \to \mathbb{R}$

let R be a rectangle in $\mathbb{R}^2$.



$R = [a,b] \times [c,d]$ $a \le x \le b$ $c \le y \le d$.

$= \{(x,y) \mid x \in [a,b],\ y \in [c,d]\}$



To define $\iint_R f(x,y)\,dA$ where $R = [a,b] \times [c,d]$

- divide $[a,b]$ into m subintervals, width $\Delta x_i = x_i - x_{i-1}$
- divide $[c,d]$ into n subintervals width $\Delta y_j = y_j - y_{j-1}$

↳ we have subrectangles R_{ij} with area $\Delta A_{ij} = \Delta x_i \Delta y_j$

- Choose a test pt. (x_{ij}^*, y_{ij}^*) in R_{ij}.
- calculate $\underbrace{f(x_{ij}^*, y_{ij}^*)}_{\text{height}} \underbrace{\Delta A_{ij}}_{\text{area}}$ = volume of column about R_{ij}, height given by f

Defn: $\underbrace{\iint_R f(x,y)\,dA}_{\text{notation}} = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_j \to 0}} \sum_{j=1}^{n} \sum_{i=1}^{m} f(x_{ij}^*, y_{ij}^*) \Delta A_{ij}$

* calculus!

Interpretation: if $f: \mathbb{R}^2 \to \mathbb{R}$, $\iint_R f(x,y)\,dA$ gives volume b/c graph of f and xy-plane.

volume below xy-plane counts neg.