Ex. $\iint_R y^2 \cos(2xy)\, dA$ $\qquad R = [0, \frac{1}{2}] \times [0, \frac{\pi}{2}]$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\frac{1}{2}} y^2 \cos(2xy)\, dx\, dy$$

let $w = 2xy$

$\frac{dw}{dx} = 2y$

$dx = \frac{dw}{2y}$

$$= \int_0^{\frac{\pi}{2}} \int_?^? \frac{y^2}{2y} \cos w\, dw\, dy$$

$$= \int_0^{\frac{\pi}{2}} \frac{y}{2} \sin(2xy) \Big|_{x=0}^{x=\frac{1}{2}} dy$$

$$= \int_0^{\frac{\pi}{2}} \frac{y}{2} \sin y - 0\, dy$$

parts $\qquad \int u\, dv = uv - \int v\, du$

$u = \frac{y}{2} \qquad dv = \sin y\, dy$

$du = \frac{1}{2} dy \qquad v = -\cos y$

$$= -\frac{y}{2}\cos y + \frac{1}{2}\int \cos y\, dy$$

$$= -\frac{y}{2}\cos y + \frac{1}{2}\sin y \Big|_0^{\frac{\pi}{2}}$$

$$= [-0 + \tfrac{1}{2}] - [-0 + 0]$$

$$= \frac{1}{2}.$$

Exercise: try

$\int_0^{\frac{1}{2}} \int_0^{\frac{\pi}{2}} y^2 \cos(2xy)\, dy\, dx$