Integration over ~~rectangular~~ general regions (in rectangular coordinates)

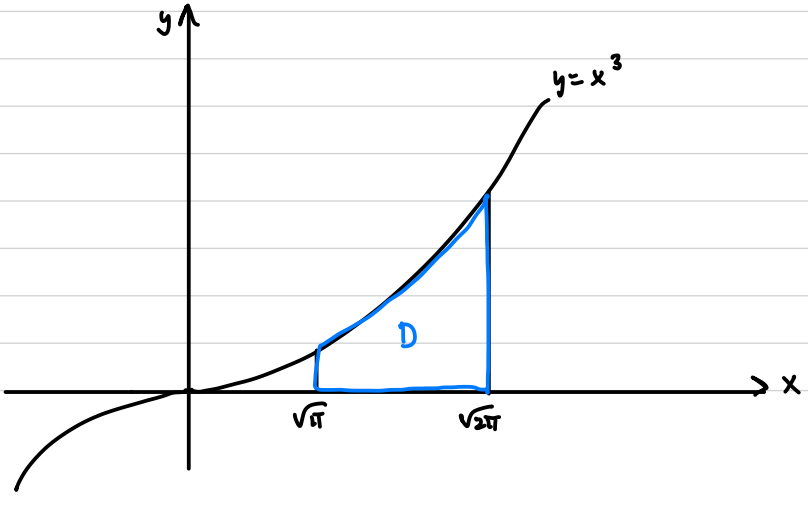
Suppose $D \subset \mathbb{R}^2$ is not a rectangle.

Q: How do you find $\iint_D f(x,y)\,dA$?

Ex $\iint_D \sin\left(\frac{y}{x}\right) dA$ where

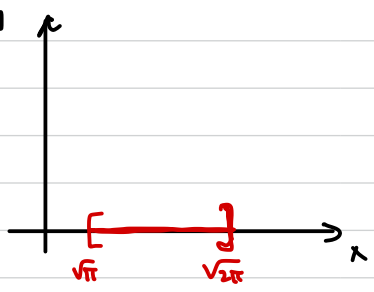
$$D = \left\{ (x,y) \;\middle|\; \sqrt{\pi} \le x \le \sqrt{2\pi},\ 0 \le y \le x^3 \right\}$$

First: DRAW D



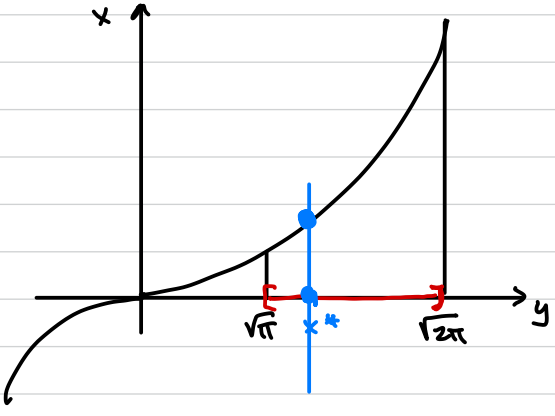
In this case:

- Project D onto x-axis:



- choose a generic pt x^* in projection.

- draw a straight line from x^* through D.



- Ask: do the same curves form boundary for any choice of x^* in projection? (Here: yes.)

- Then: for any x^*, area slice at x^* is $\int_{y=0}^{y=(x^*)^3} f(x^*, y)\, dy$ (upper bdry: $y=(x^*)^3$; lower bdry: $y=0$)

- Finally: integrate in x-direction: $\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^3} f(x,y)\, dy\, dx$