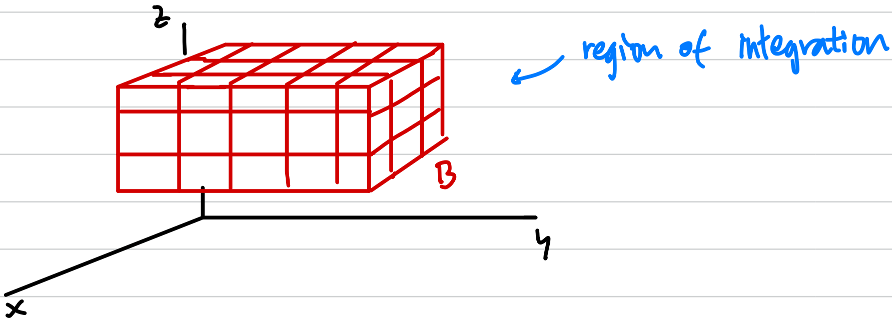
Triple Integrals

Define a triple integral of $f(x,y,z)$ over a box similarly to a double integral of $f(x,y)$ over a rectangle.



- Divide B into subboxes B_{ijk} of volume $\Delta V_{ijk} = \Delta x_i \Delta y_j \Delta z_k$

- Choose a test point $(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)$ in each B_{ijk}

- Define $$\iiint_B f(x,y,z)\,d\overset{\text{volume}}{V} = \lim_{\Delta V_{ijk}\to 0} \underbrace{\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} \underbrace{f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)}_{\text{function}}\underbrace{\Delta V_{ijk}}_{\text{domain}}}_{\text{Riemann sum}}$$

* Calculus

An interpretation: If f measures density and ΔV measures volume, $\iiint_B f\,dV$ gives mass of B.

Note: $\iint_D 1\,dA$ = area D

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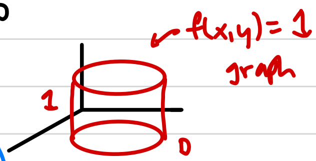
= vol solid

= (height)(area base)

= 1 (area base)

= area base

(same number... ignoring units)



$\iiint_B 1\,dV$ = volume B

similar argument for volume.

Thm If f is cts on the box

$$B = [a,b] \times [c,d] \times [r,s], \text{ then}$$

$$\iiint_B f(x,y,z)\,dV = \int_a^b \int_c^d \int_r^s f(x,y,z)\,dz\,dy\,dx$$

notation... limit of Riemann sums

... or any other order of iterated integration

Note: six orders of integration...