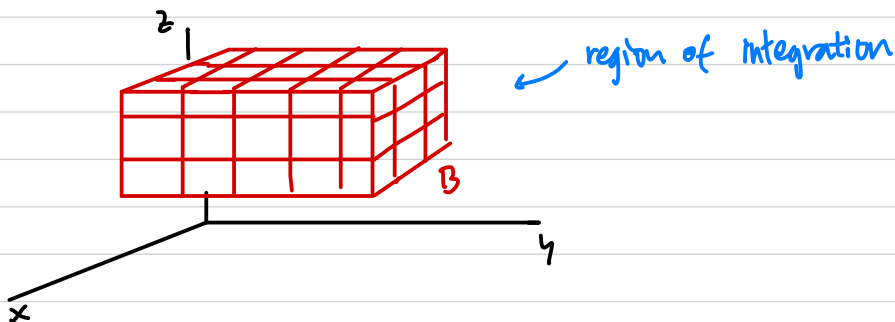


Triple Integrals

Define a triple integral of $f(x, y, z)$ over a box similarly to a double integral of $f(x, y)$ over a rectangle.



• Divide B into subboxes B_{ijk} of volume $\Delta V_{ijk} = \Delta x_i \Delta y_j \Delta z_k$

• Choose a test point $(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)$ in each B_{ijk}

• Define $\iiint_B f(x, y, z) dV = \lim_{\Delta V_{ijk} \rightarrow 0} \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^n f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*) \Delta V_{ijk}$

Annotations:

- A blue arrow points from the word "volume" to ΔV_{ijk} .
- A red arrow points from the word "Calculus" to the asterisks in the test point coordinates.
- A blue wavy line underlines the function $f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)$, with a blue arrow pointing to it from the word "function".
- A blue wavy line underlines the domain ΔV_{ijk} , with a blue arrow pointing to it from the word "domain".
- A blue wavy line underlines the entire summation expression, with the label "Riemann sum" below it.

An interpretation: If f measures density and ΔV measures volume, $\iiint_B f dV$ gives mass of B .

Note: $\iint_D 1 dA = \text{area } D$

$\iiint_B 1 dV = \text{volume } B$

$\iint_D 1 dA$

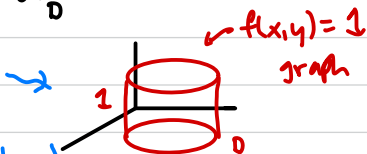
= vol solid

= (height)(area base)

= 1 (area base)

= area base

(same number... ignoring units)



similar argument for volume.

Thm If f is cts on the box

$B = [a, b] \times [c, d] \times [r, s]$, then

$\iiint_B f(x, y, z) dV = \int_a^b \int_c^d \int_r^s f(x, y, z) dz dy dx$

notation...
limit of Riemann sums

... or any other order of iterated integration

↳ Note: six orders of integration...