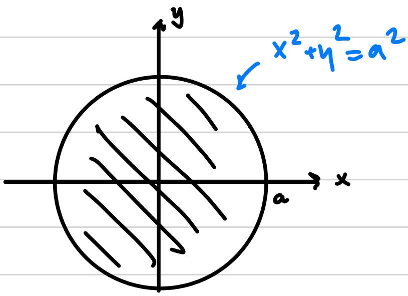
Change of Variables

Motivating example: Compute area of circle of radius a.



ie. compute: $\iint_D 1\, dA$

Recall rectangular coordinates:

$$= \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 1 \, dy \, dx$$

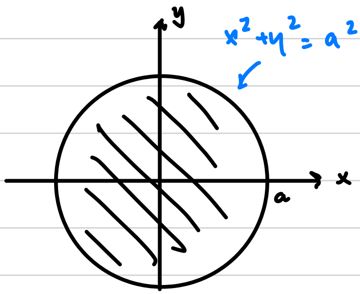
$$= \int_{-a}^{a} 2\sqrt{a^2-x^2}\, dx$$

need trig sub $\quad x = a\sin\theta$

$dx = a\cos\theta \, d\theta$

etc...

Notice:



in terms of polar coords, D is:

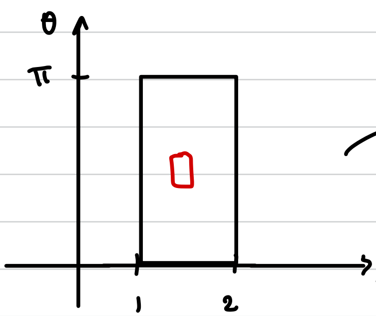
$$0 \le \theta \le 2\pi$$
$$0 \le r \le a$$

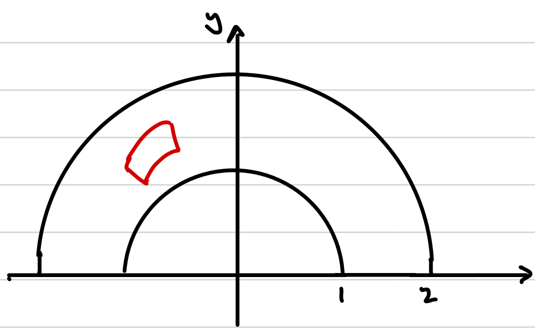
GOAL: change variables of integral to polar coords.

We will see: integral in polar coords is: $\displaystyle\int_0^{2\pi}\int_0^a 1\, \underbrace{r}_{*}\, dr\, d\theta$

↳ * BIG IDEA *: Need to account for area/volume distortion when you change coord systems.

Ex.





Area gets distorted by transformation map.

Return to this in a bit...

We'll see: If R is a region in $\mathbb{R}^2$ described in polar coordinates by $a \leq r \leq b$ and $\alpha \leq \theta \leq \beta$, then

$$\iint_R f(x,y)\,dy\,dx = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,\underbrace{r}_{*}\,dr\,d\theta.$$