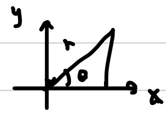
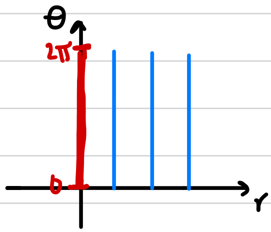
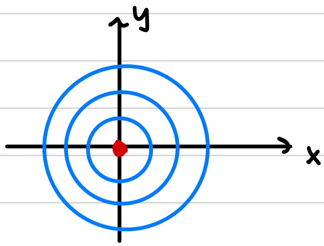
Defn: A change of coordinates from (u,v) to (x,y) is a differentiable mapping $T: \underset{(u,v)}{\mathbb{R}^2} \longrightarrow \underset{(x,y)}{\mathbb{R}^2}$ which is one-to-one almost everywhere.



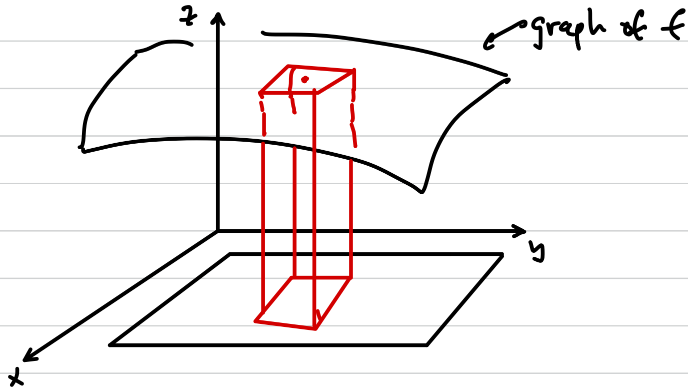
Important example: $T(r,\theta) = (\underset{x(r,\theta)}{r\cos\theta}, \underset{y(r,\theta)}{r\sin\theta})$ ← polar coords.

(other examples, in $\mathbb{R}^3$: cylindrical, spherical)





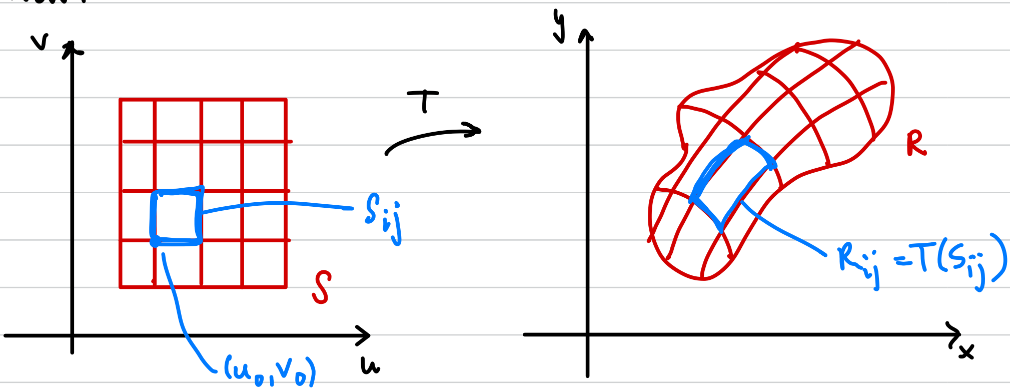
Now, recall double integration!



To find $\iint_R f(x,y)\,dA$, we summed volumes of bars:

$$V_{bar} = f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

Now:



Sps $T: S \to R$. $\quad T(u,v) = (x(u,v), y(u,v))$

change of variables function.

Sps we know area $S_{ij} = \Delta u \Delta v$.

Q: What is corresponding area ΔA of $R_{ij} = T(S_{ij})$