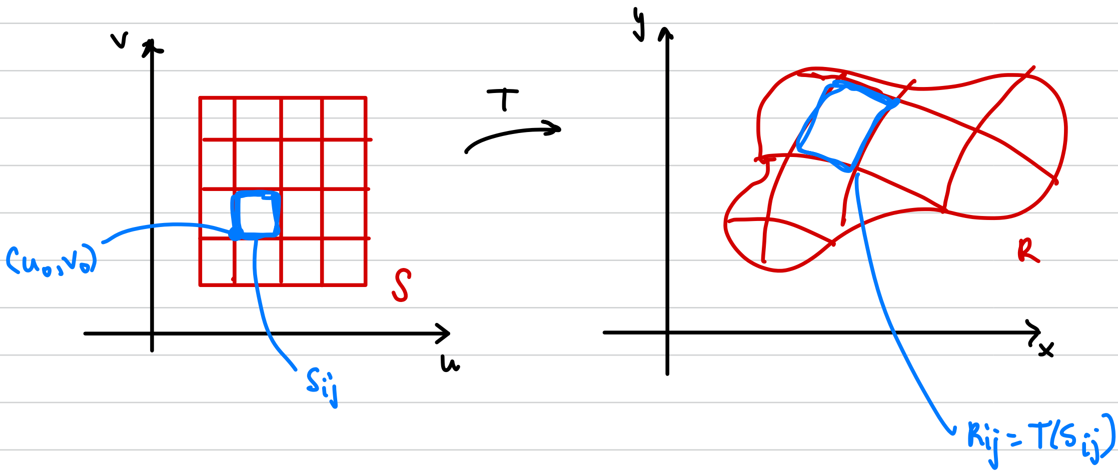
Sps T is a change of coordinates and area $S_{ij} = \Delta u \Delta v$



Q: What is corresponding area of R_{ij}, aka $T(S_{ij})$?

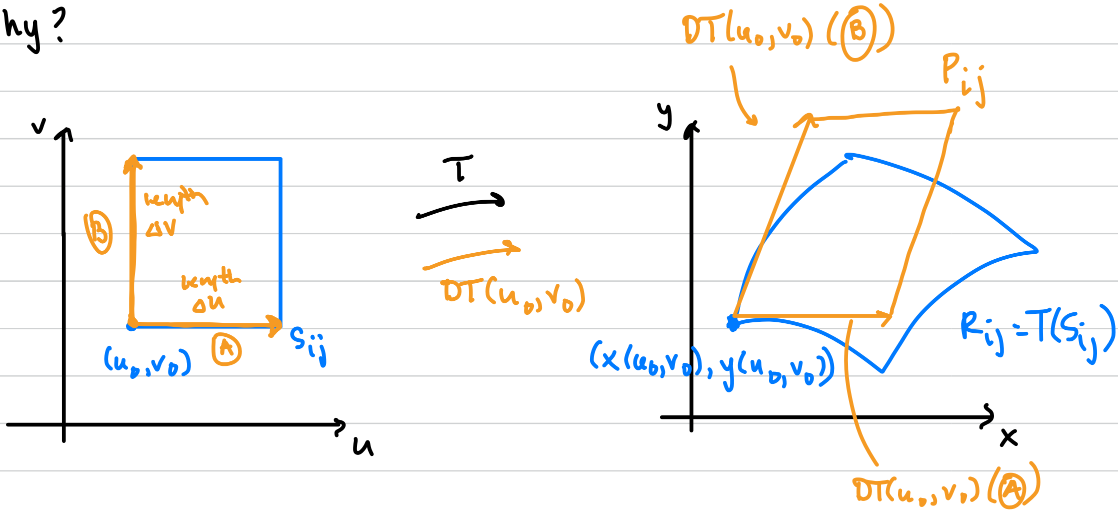
ΔA.

Claim: $\underbrace{\Delta A}_{\text{area } T(S_{ij}) \text{ (aka } R_{ij})} \approx \underbrace{|\det \underbrace{DT(u_0, v_0)}_{\text{square matrix}}|}_{\text{measurement of distortion}} \underbrace{\Delta u \Delta v}_{\text{area } S_{ij}}$

approximation better as area $(S_{ij}) \to 0$

WTS: $\Delta A \approx |\det DT(u_0, v_0)| \, \Delta u \Delta v$

Why?

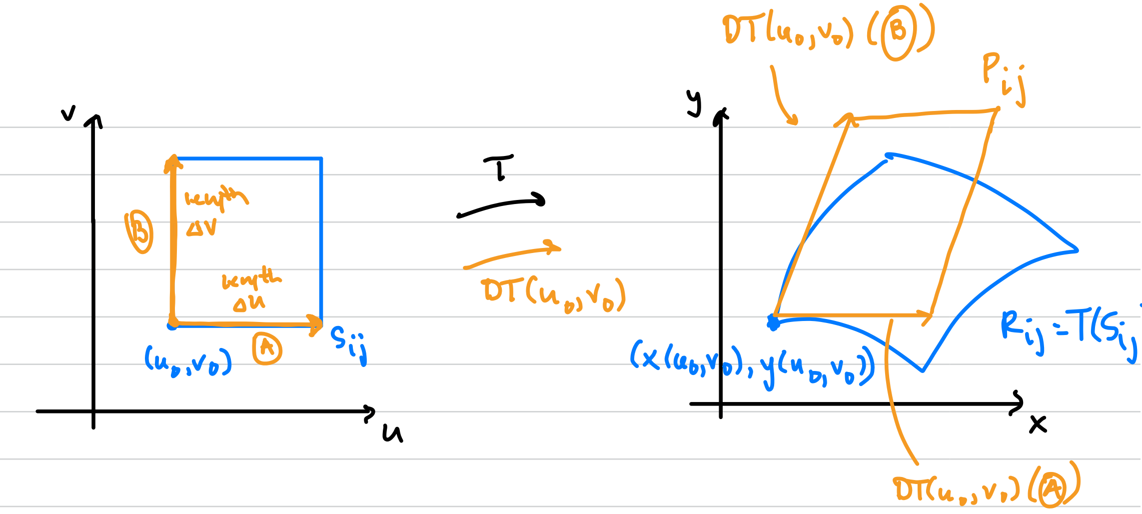


Recall: $DT(u_0, v_0) : \{\text{vectors based at } (u_0, v_0)\} \longrightarrow \{\text{vectors based at } (x(u_0,v_0), y(u_0,v_0))\}$

Also: Linear transformations map parallelograms to parallelograms.

Finally: the area distortion of a linear map is given by

|determinant|. ← abs. value



So here:

Area S_{ij} (i.e. parallegram formed by A and B) is:

$$\text{area } S_{ij} = \Delta u \Delta v$$

$P_{ij} = DT^{(u_0,v_0)}(S_{ij})$ is a parallelogram based at $(x(u_0,v_0), y(u_0,v_0))$

Area $P_{ij} \underset{*}{=} |\det DT(u_0,v_0)|$ area $S_{ij} = |\det DT(u_0,v_0)| \Delta u \Delta v$

Finally: area $P_{ij} \approx$ area R_{ij} when $\Delta u, \Delta v$ are small.

Thus: area $R_{ij} \approx |\det DT(u_0,v_0)| \Delta u \Delta v$

$T(S_{ij})$