Change of variables for triple integrals:

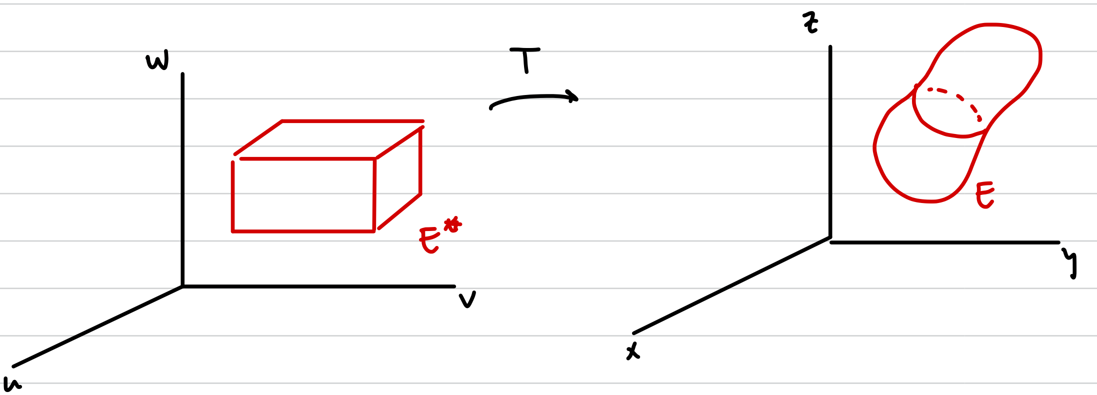
Same reasoning shows that if

$$T : \mathbb{R}^3 \to \mathbb{R}^3$$

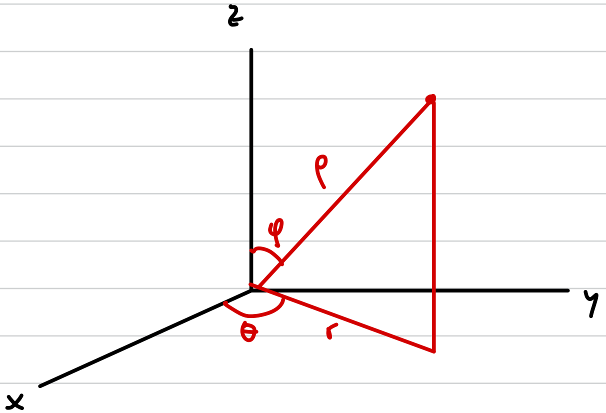
is a change of coordinates transformation, then

$$\iiint_E f(x,y,z)\,dx\,dy\,dz$$

$$= \iiint_{E^*} f(x(u,v,w), y(u,v,w), z(u,v,w)) \underbrace{\left|\det DT(u,v,w)\right|}_{\text{distortion factor}}\, du\,dv\,dw$$



Recall: cylindrical and spherical coordinates:



$\rho \geq 0$

$r \geq 0$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi$

cylindrical: $T(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$

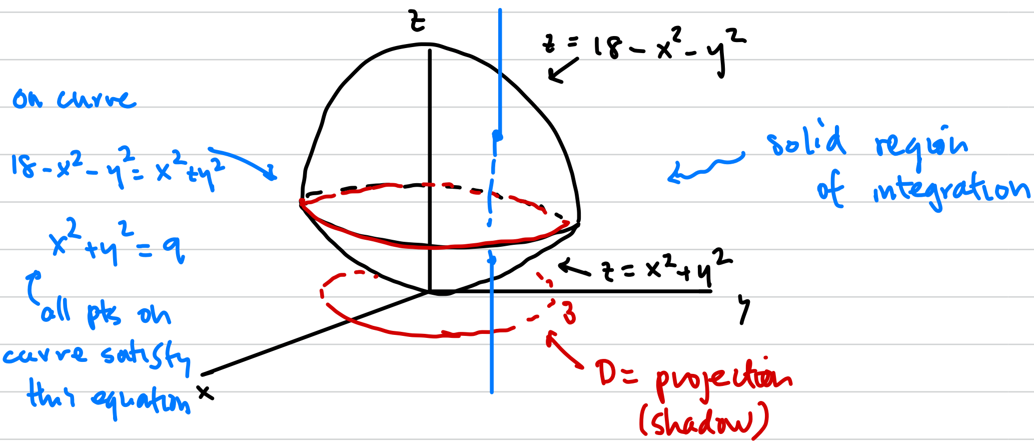
$$|\det DT(r,\theta,z)| = \left|\det\begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| = r$$

cylindrical distortion factor

<u>Ex</u> Set up integral of $f(x,y,z) = ze^{x^2+y^2}$

over the region between the paraboloids

$z = 18 - x^2 - y^2$ and $z = x^2+y^2$.



Boundary of D is projection of C onto xy-plane.

upper surface $z = 18-x^2-y^2$

$$\int_0^{2\pi}\int_0^3\int_{r^2}^{18-r^2} z e^{r^2}\, r\ \ dz\,dr\,d\theta.$$

lower surface $z = x^2+y^2$