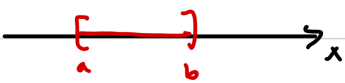
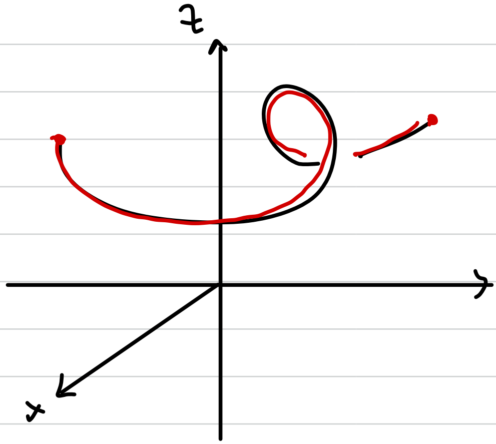
Line Integrals

↳ single variable integrals, but over curves in $\mathbb{R}^2$ (or $\mathbb{R}^3$) as opposed to interval in $\mathbb{R}$.



single variable calculus

and now →



line integrals in multivariable calculus

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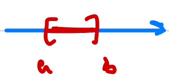
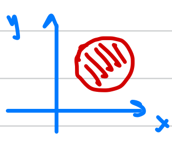
we've generalized integration $f: \mathbb{R} \to \mathbb{R}$
to $f: \mathbb{R}^2 \to \mathbb{R}$ and
$f: \mathbb{R}^3 \to \mathbb{R}$



single variable

$\int f\,dx$

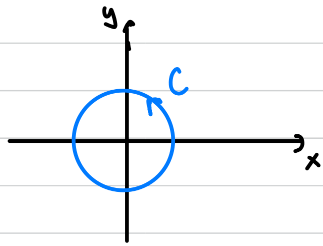


$\iint f\,dA$

$\iiint f\,dV$

Suppose C is a curve in $\mathbb{R}^n$ given by $\bar{x}$:

e.g. $\bar{x}(t) = (\underbrace{\cos t}_{x(t)}, \underbrace{\sin t}_{y(t)}) \quad 0 \le t \le 2\pi$



Now, sps $f: \mathbb{R}^n \to \mathbb{R}$.

Then $f(\bar{x}(t))$ is a restrict of f to the curve given by $\bar{x}(t)$.

e.g. $f(x,y) = x^2 y$

then $f(\bar{x}(t)) = (\cos t)^2 (\sin t)$

Any choice of t gives the value of f at the corresponding point on C.

Goal: Integrate (scalar) function f along C.