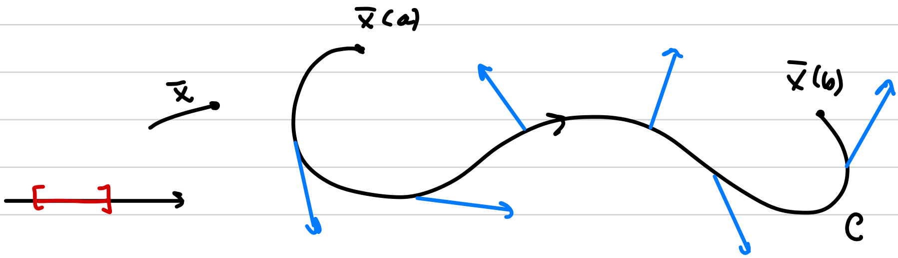
line integral of a scalar-valued function

We know how to compute $\int_C f \, ds$.

Now: line integrals of vector fields.

Defn Sps C is a curve in $\mathbb{R}^n$ given by $\bar{x}(t)$, $a \le t \le b$.

If $\bar{F}$ is a vector field along C



define

$$\underbrace{\int_C \bar{F} \cdot d\bar{s}}_{\text{notation}} = \underbrace{\int_a^b \bar{F}(\bar{x}(t)) \cdot \bar{x}'(t) \, dt}_{\text{how to compute.}}$$

Ex. $\bar{F}(x,y) = (x^2y, 2x-y)$ along $\bar{x}(t) = (t^2, t)$ $\quad 0 \le t \le 1$

$\bar{F}(\bar{x}(t)) = \left((t^2)^2(t), 2t^2 - t\right) = (t^5, 2t^2 - t)$

$\bar{x}'(t) = (2t, 1)$

$$\int_C \bar{F} \cdot d\bar{s} = \int_0^1 (t^5, 2t^2 - t) \cdot (2t, 1)\, dt = \int_0^1 2t^6 + 2t^2 - t \, dt$$

* pull back by $\bar{x}$, get everything in terms of t.