Q: What does $\int_C \bar{F} \cdot d\bar{s}$ measure?

A: Work.

↳ i.e. the tendency of $\bar{F}$ to push a particle along C.

Why?

First: rewrite $\int_C \bar{F} \cdot d\bar{s}$:

$$\int_C \bar{F} \cdot d\bar{s} = \int_a^b \underbrace{\bar{F}(\bar{x}(t)) \cdot \bar{x}'(t)}_{\text{scalar for each } t} \, dt$$

$$= \int_a^b \overbrace{\bar{F}(\bar{x}(t)) \cdot \underbrace{\frac{\bar{x}'(t)}{\|\bar{x}'(t)\|}}_{}}^{\bar{F}\cdot\bar{T} \text{ .... a scalar-valued function}} \|\bar{x}'(t)\| \, dt$$

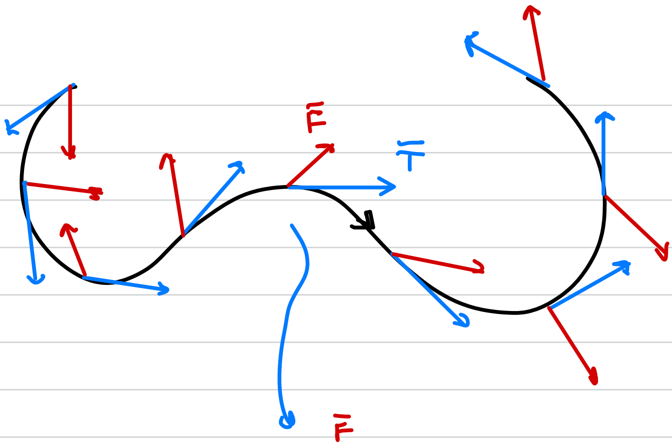
↳ call this $\bar{T}$... unit tangent vector to C at $\bar{x}(t)$.

defn of $\int_C f \, ds$

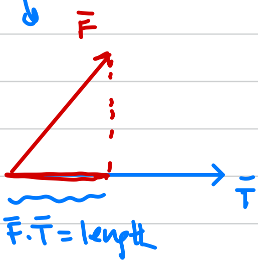
$$= \int_C (\bar{F} \cdot \bar{T}) \, ds$$

* MAKE NOTE! *

* Alternate form of $\int_C \bar{F} \cdot d\bar{s}$



Consider $\bar F \cdot \bar T$:



Recall projection : $\text{proj}_{\bar T} \bar F = \underbrace{\left(\frac{\bar F \cdot \bar T}{\|\bar T\|} \right)}_{\text{length}} \underbrace{\frac{\bar T}{\|\bar T\|}}_{\text{direction}}$

But : $\|\bar T\| = 1$.

So at each point, $\bar F \cdot \bar T$ measures signed length of projection of $\bar F$ onto $\bar T$.... how much of $\bar F$ pointing in direction of T.

Thus:

from before

$$\int_C \bar{F}\cdot d\bar{s} = \int_C \bar{F}\cdot\bar{T}\,ds$$

= "sum" of $\bar{F}\cdot\bar{T}$ along C

= cumulative tendency of $\bar{F}$ to push in dir. of C

= work.

So: to compute work done by force field $\bar{F}$ in moving a particle along C, compute $\int_C \bar{F}\cdot d\bar{s}$.

Note: If $-C$ is same curve as C, traversed in opposite direction,

$$\int_{-C} \bar{F}\cdot d\bar{s} = -\int_C \bar{F}\cdot d\bar{s}$$

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