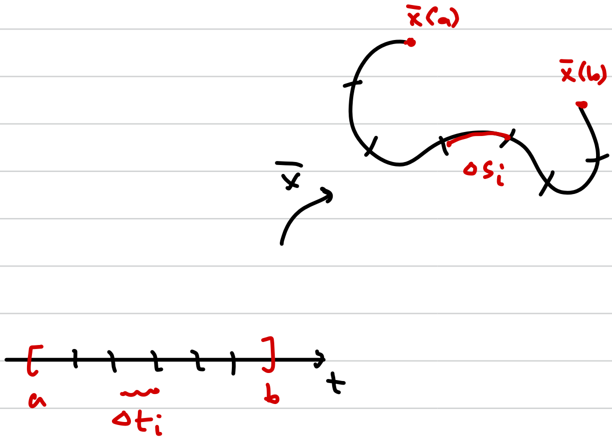
Finally, some notation:



Recall defn of $\int_C f\,ds$:

$$\int_C f\,ds = \lim_{\Delta t_i \to 0} \sum_{i=1}^{n} f(\bar{x}(t_i^*))\,\Delta s_i$$

*

We could define instead:

$$\int_C f\,dx = \lim_{\Delta t_i \to 0} \sum_{i=1}^{n} f(\bar{x}(t_i^*))\,\Delta x_i$$

*measure change of x coord. in i^{th} subinterval of C

or

$$\int_C f\,dy = \lim_{\Delta t_i \to 0} \sum_{i=1}^{n} f(\bar{x}(t_i^*))\,\Delta y_i$$

change in y-coord.

Sps $\bar{x}(t) = (x(t), y(t))$.

To compute $\int_C f\,dx$, note that $\overset{x'(t)}{\frac{dx}{dt}} \approx \frac{\Delta x}{\Delta t}$ so: $\Delta x \approx \underbrace{x'(t)}_{\text{distortion factor for x measurement}} \Delta t$

Thus, $\int_C f\,dx = \int_a^b f(\bar{x}(t))\,\overbrace{x'(t)}^{*}\,dt$

pull back by $\bar{x}$ (param) to t-axis, accounting for distortion

and similarly, $\int_C f\,dy = \int_a^b f(\bar{x}(t))\,\overbrace{y'(t)}^{*}\,dt$

<u>Ex</u> $\int_C x^3 y\,dx + 2x\,dy$ over curve (t^5+2t, t^4) $1 \le t \le 2$

$x'(t) = 5t^4 + 2$ $\qquad$ $y'(t) = 4t^3$

$$\int_1^2 \underbrace{(t^5+2t)^3(t^4)}_{\text{from } x^3y}\underbrace{(5t^4+2)}_{x'(t)} + \underbrace{2(t^5+2t)}_{\text{from } 2x}\underbrace{(4t^3)}_{y'(t)}\,dt$$

vector field

component functions

Now, suppose $\bar{F}(x,y,z) = (M(x,y,z),\ N(x,y,z),\ P(x,y,z))$

and $\bar{x}(t) = (x(t), y(t), z(t))$ $\quad a \le t \le b$

Then $\int_C \bar{F} \cdot d\bar{s} = \int_a^b F(\bar{x}(t)) \cdot \bar{x}'(t)\,dt$

defn of $\int_C \bar{F} \cdot d\bar{s}$

$$= \int_a^b (M(\bar{x}(t)), N(\bar{x}(t)), P(\bar{x}(t))) \cdot (x'(t), y'(t), z'(t))\,dt$$

do the dot product

$$= \int_a^b M(\bar{x}(t))\,x'(t) + N(\bar{x}(t))\,y'(t) + N(\bar{x}(t))\,z'(t)\,dt$$

components of $\bar{F}$ *

defn of $\int_C f\,dx$ etc.

$$= \int_C M\,dx + N\,dy + P\,dz$$

MAKE NOTE: alternate notation $\int_C \bar{F} \cdot d\bar{s}$

Summary of $\int_C \bar{F} \cdot d\bar{s}$: $\quad \bar{F} = (M, N, P) \quad \bar{x}(t) = (x(t), y(t), z(t))$

components of $\bar{F}$.

$$\underbrace{\int_C \bar{F} \cdot d\bar{s}}_{\text{notation}} = \underbrace{\int_a^b \bar{F}(\bar{x}(t)) \cdot \bar{x}'(t)\,dt}_{\text{how to compute}} = \underbrace{\int_C (\bar{F} \cdot \bar{T})\,ds}_{\text{alt notation, good for interpretation}} = \underbrace{\int_C M\,dx + N\,dy + P\,dz}_{\text{another alt notation}}$$

Summary of orientation: $\int_{-C} f\,ds = \int_C f\,ds$.

but $\int_{-C} \bar{F} \cdot d\bar{s} = -\int \bar{F} \cdot d\bar{s} \qquad \int_{-C} f\,dx = -\int_C f\,dx \qquad \int_{-C} f\,dy = -\int_C f\,dy$