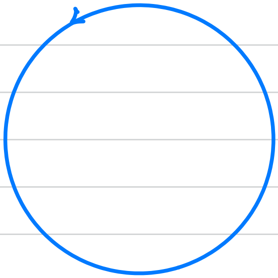
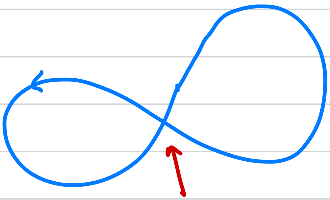
Green's Thm

A curve C is <u>simple</u> if it has no self-intersections.



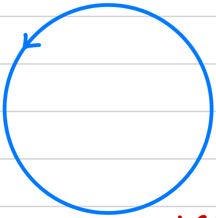
vs.



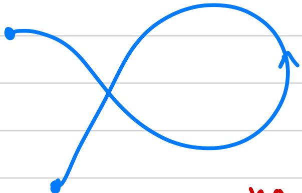
no

yes

It is <u>closed</u> if it has no endpoints.



vs.



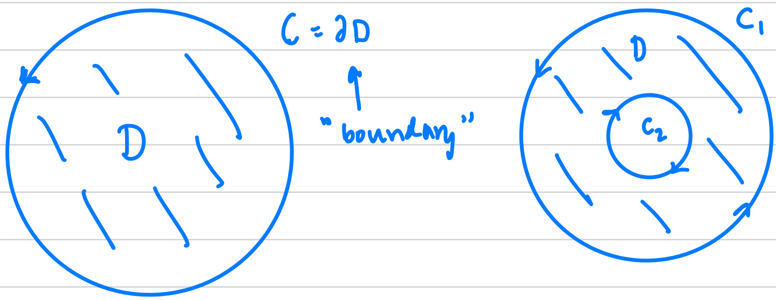
yes

no

Sps D is bounded by a simple closed curve C.

C has <u>positive orientation</u> relative to D

if D is on your left as you traverse C.



<u>Thm</u> (Green's Thm)

Let D be a closed, bounded region in $\mathbb{R}^2$ * whose boundary C is made up of a finite number of simple closed curves. Sps. C has positive orientation rel. to D.

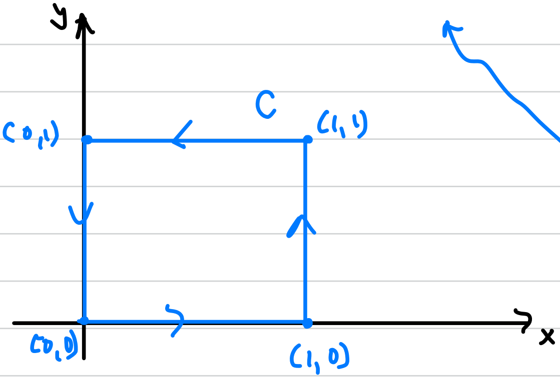
If $M(x,y)$ and $N(x,y)$ have continuous partial derivatives in D then

$F = (M,N)$

$$\int_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Note: sometimes $\int_C$ written $\oint_C$ when C closed.

Ex: Evaluate $\oint_C x^3 y\,dx + e^y\,dy$ over curve C.



Note: this is $\int_C \bar{F}\cdot ds$ where $\bar{F} = (x^3y, e^y)$.

↳ 4 components to C = BIG PAIN.

check: orientation of C pos. rel to interior ✓

By Green's Thm:

$$\int_0^1\int_0^1 \overset{\partial N/\partial x}{0} - \overset{\partial M/\partial y}{x^3}\,dy\,dx$$

$$\int_0^1 -x^3y\Big|_0^1\,dx$$

$$= \int_0^1 -x^3\,dx$$

$$= -\frac{1}{4}x^4\Big|_0^1 = -\frac{1}{4}.$$