

Big idea: Generalized Stoke's Thm (exterior derivative  $d$ )  
(differential form  $\omega$ )

$$\overset{\text{exterior derivative}}{\int_R} d\omega = \int_{\partial R} \omega.$$

$\uparrow$   
boundary

Compare with Fundamental Theorem of Calculus:

$$\int_a^b F'(t) dt = F(b) - F(a)$$

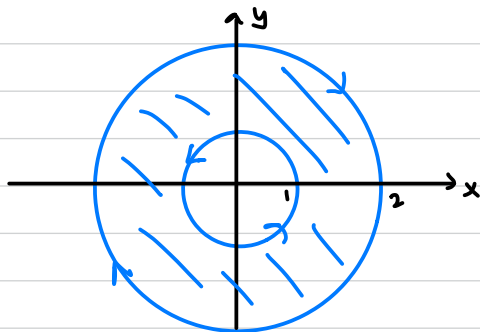
$$\underbrace{dF}_{\int_{[a,b]} F'} = \underbrace{\int_{\text{bdry}} F}_{\text{0-dim'l integral over bdry}}$$

$$F = \omega$$

$$[a, b] = R$$

$$\text{deriv.} = \text{exterior deriv.}$$

Ex Find  $\oint_C xy^3 dx + yx^3 dy$  where  $C$  is the boundary of the region:



$$\vec{F} = (xy^3, yx^3)$$

with negative orientation relative to  $D$ .

b/c of neg orientation  $\rightarrow - \iint_D \left( 3x^2y \overset{\frac{\partial N}{\partial x}}{\leftarrow} - 3xy^2 \overset{\frac{\partial M}{\partial y}}{\leftarrow} \right) dA$   $\leftarrow$  (at this stage, done w/ Greens...

now just compute)

(polar conversion

$$- \int_0^{2\pi} \int_1^2 \left[ 3(r \cos \theta)^2 (r \sin \theta) - 3(r \cos \theta)(r \sin \theta)^2 \right] r dr d\theta$$

$\rightarrow$  distortion!

= etc.