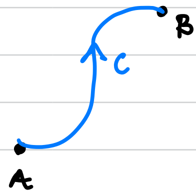
By FTLI we know conservative vector fields

are nice:

$$\int_c \nabla f \cdot d\bar{s} = f(B) - f(A)$$



$\hookrightarrow$ $\int_c \nabla f \cdot d\bar{s}$ is independent of path.

Q: how can we tell if $\bar{F}$ is conservative?

Sps $\bar{F} = (M, N)$ is conservative

Then $M = f_x$, $N = f_y$ for some f.

By equality of mixed partials, it must be that

$$N_x = (f_y)_x = (f_x)_y = M_y.$$

So: if $N_x \neq M_y$, $\bar{F}$ is not conservative.

More generally, sps $\bar{F} = (M, N, R)$ is conservative.

$$\text{Then } \bar{F} = (\overset{M}{f_x}, \overset{N}{f_y}, \overset{R}{f_z})$$

$$\text{So } \operatorname{curl} \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_x & f_y & f_z \end{vmatrix}$$

$$= (f_{zy} - f_{yz}, f_{xz} - f_{zx}, f_{yx} - f_{xy}) = \bar{0}$$

(recall: $\operatorname{curl}(\nabla f) = \bar{0}$)

So: if $\operatorname{curl} \bar{F} \neq \bar{0}$, $\bar{F}$ is not conservative.

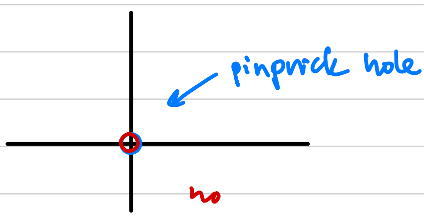
In general, curl $\bar{F} = \bar{0} \not\Rightarrow$ $\bar{F}$ conservative.

BUT: converse holds on simply connected region in $\mathbb{R}^3$ (or $\mathbb{R}^2$).

A simply connected region is

connected:

and has no holes:



<u>Thm</u> If $\bar{F} = (M, N, P)$ (or $\bar{F} = (M, N)$) is a vector field on a simply connected region in $\mathbb{R}^3$ (or $\mathbb{R}^2$) such that M, N, P have continuous partial derivatives then

$\bar{F}$ is conservative $\Longleftrightarrow$ curl $\bar{F} = \bar{0}$ (or $M_y = N_x$)