

Sps. you know (or suspect) that $\vec{F}$ is conservative. How do you find potential function f (so you can use FTLI)?

EX $\vec{F} = (2y^3 + 2x)\vec{i} + (6xy^2 + z)\vec{j} + (y + 3z^2)\vec{k} = \nabla f$. Find f .

$\uparrow$ $\uparrow$ $\uparrow$
 $\textcircled{1} f_x$ $\textcircled{2} f_y$ $\textcircled{3} f_z$

$\textcircled{1} f_x = 2y^3 + 2x$

int w.r.t. $x \Rightarrow f(x, y, z) = 2xy^3 + x^2 + \underbrace{g(y, z)}_{+C \text{ for } x}$ $\textcircled{A}$

diff w.r.t. y $\left\{ \begin{array}{l} f_y = 6xy^2 + g_y \\ \text{But by } \textcircled{2} \quad f_y = 6xy^2 + z \end{array} \right. \Rightarrow g_y(y, z) = z \Rightarrow g(y, z) = yz + h(z)$ $\textcircled{B}$

$+C \text{ for } y$
 $\downarrow$

$\textcircled{A}, \textcircled{B} \Rightarrow f(x, y, z) = 2xy^3 + x^2 + yz + h(z)$ $\textcircled{C}$

diff. w.r.t. z $\left\{ \begin{array}{l} f_z = y + \frac{dh}{dz} \\ \text{But by } \textcircled{3} \quad f_z = y + 3z^2 \end{array} \right. \Rightarrow \frac{dh}{dz} = 3z^2 \Rightarrow h(z) = z^3 + C$ $\textcircled{D}$

$\textcircled{C}, \textcircled{D} \Rightarrow \underline{f(x, y, z) = 2xy^3 + x^2 + yz + z^3 + C.}$