MATH 226 Differential Equations

Assignment 1

Due: Friday, February 14

Reading

Read carefully Section 1.1 “Mathematical Models and Solutions” in our text *Differential Equations* by Brannan and Boyce.

Writing

Integration Review

Our textbook comes with an online integration review handout, complete with review of basic techniques and practice problems. Integration techniques will be necessary when using certain solution methods. The handout can be accessed online on the publisher's website for the text (**http://www.wiley.com/WileyCDA/WileyTitle/productCd-EHEP003244.html**) or by following the link below or on the course website:

Integration Review Complete the following practice problems in the handout above. Complete the feedback problems on separate sheets of paper, as they will be handed in for feedback.

Practice Problems: 1, 3, 4, 5, 6, 7, 10, 11, 13, 14

Feedback Problems: 4, 10, 11

The two solved integration problems below illustrate the level of detail and use of complete sentences we expect to see in your write ups of solutions to the Feedback Problems:

9. $\int 3x^2\sqrt{3x}\,dx$

Solution: Rewrite the integrand as $3x^2\sqrt{3x}=3\sqrt{3}x^2\sqrt{x}=3\sqrt{3}x^2x^{1/2}=3\sqrt{3}x^{2+1/2}=3\sqrt{3}x^{5/2}$.

Using the fact that the integral of a constant times a function is the constant times the integral of the function and the Power Rule: $\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$ with $n=\frac{5}{2}$, we have

$$\int 3x^2\sqrt{3x}\,dx=\int 3\sqrt{3}x^{5/2}\,dx=3\sqrt{3}\int x^{5/2}\,dx=3\sqrt{3}\frac{x^{7/2}}{7/2}+C=\frac{2}{7}3\sqrt{3}x^{7/2}+C=\frac{6}{7}\sqrt{3}x^{7/2}+C$$

13. Use Partial Fraction Decompostion:

$$\frac{1}{x\left(x^2+4\right)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}=\frac{A(x^2+4)+x(Bx+C)}{x\left(x^2+4\right)}=\frac{Ax^2+4A+Bx^2+Cx}{x\left(x^2+4\right)}=\frac{(A+B)x^2+Cx+4A}{x\left(x^2+4\right)}$$

Equating Numerators, we have: $A+B=0, C=0, 4A=1$ so $A=1/4$ and $B=-1/4$.

Thus $\int\frac{1}{x\left(x^2+4\right)x}\,dx=\int\left(\frac{1}{4}\frac{1}{x}-\frac{(1/4)x}{x^2+4}\right)dx=\frac{1}{4}\int\frac{1}{x}\,dx-\frac{1}{4}\int\frac{x}{x^2+4}\,dx=\frac{1}{4}\ln|x|-\frac{1}{8}\ln\left|x^2+4\right|+C$

The second integral $\int\frac{x}{x^2+4}\,dx$ is solved by using the substitution $u=x^2+4$ so $du=2x\,dx$ so $x\,dx=\frac{1}{2}du$.

Then $\int\frac{x}{x^2+4}\,dx=\int\frac{1}{2}\frac{1}{u}\,du=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln\left|x^2+4\right|+C$.