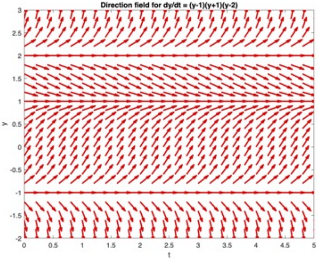
MATH 226:Differential Equations



Class 4:February 17, 2025

Notes on Assignment 2

Assignment 3

Direction Field for $y' = f(t,y)$

Qualitative Analysis of Autonomous Differential Equation

$$\frac{dy}{dt} = f(y)$$

1. **Find Equilibrium Solutions** ($f(y) = 0$)
2. **Create Phase Line**
   Determine when $f(y) > 0$ and where $f(y) < 0$
   Label with arrows.
3. **Classify Equilibrium Solutions**
   Asymptotically Stable: $\rightarrow \bullet \leftarrow$
   Semistable: $\rightarrow \bullet \rightarrow$ or $\leftarrow \cdot \leftarrow$
   Unstable: $\leftarrow \bullet \rightarrow$

Asymptotically Stable	Semistable	Unstable
$\downarrow$	$\downarrow\uparrow$	$\uparrow$
$\bullet$	$\bullet\ \bullet$	$\bullet$
$\uparrow$	$\downarrow\ \uparrow$	$\downarrow$

4. **Sketch Solutions**
   Increasing, Decreasing, Concavity

Determining Concavity of y as a Function of t

$$y'(t) = f(y(t) \text{ or more simply } y' = f(y)$$

- Use Second Derivative:

$$y''(t) = f'(y(t) \times y'(t) = \frac{df}{dy} \times \frac{dy}{dt} = \frac{df}{dy} \times f(y)$$

- Use Graph of $f(y)$ as a function of y

Determining Concavity of y as a Function of t

Example: $y' = (y-1)(y+1)(y-2) = (y^2-1)(y-2)$

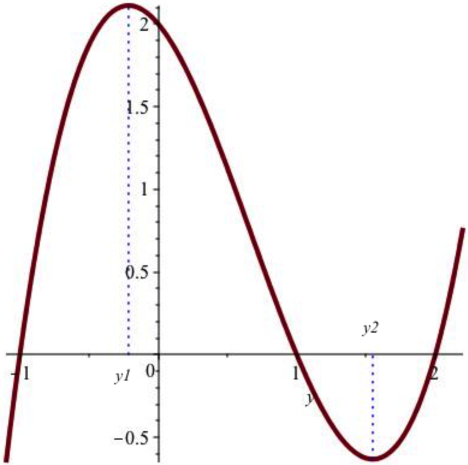
- Use Second Derivative:

$$y''(t) = f'(y(t) \times y'(t) = \frac{df}{dy} \times \frac{dy}{dt} = \frac{df}{dy} \times f(y)$$

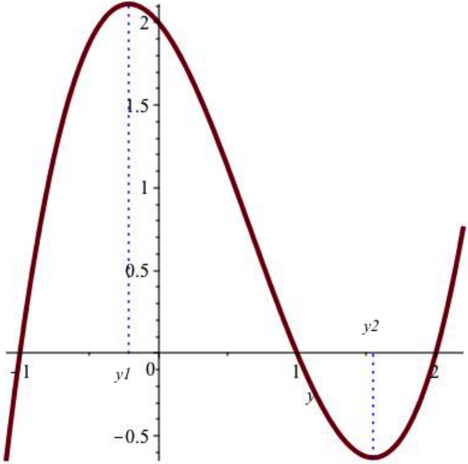
$$y'' = [3y^2 - 4y - 1][(y^2-1)(y2)]$$

- **Use Graph of $f(y)$ as a function of y**

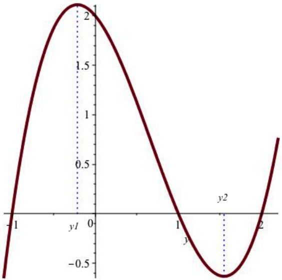
Graph of $f(y) = (y^2 - 1)(y - 2)$ as a function of y



Graph of $f(y) = (y^2 - 1)(y - 2)$ as a function of y



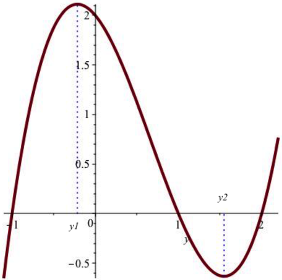
$y1 \sim -.215, y_2 \sim 1.549$



Since $y'(t) = f(y(t))$, we have $y''(t) = \frac{df}{dy}\frac{dy}{dt} = f''(y(t))y'(t)$

Interval of y values	Behavior of Derivative as function of y	$y''(t) = \frac{dy'}{dt} = \frac{dy'}{dy}\frac{dy}{dt}$
$y < -1$	Negative and Increasing	$(+)(-) = -$
$-1 < y < r_1$	Positive and Increasing	$(+)(+) = +$
$r_1 < y < 1$	Positive and Decreasing	$(-)(+) = -$
$1 < y < y_2$	Negative and Decreasing	$(-)(-) = +$
$y_2 < y < 2$	Negative and Increasing	$(+)(-) = -$
$y > 2$	Positive and Increasing	$(+)(+) = +$

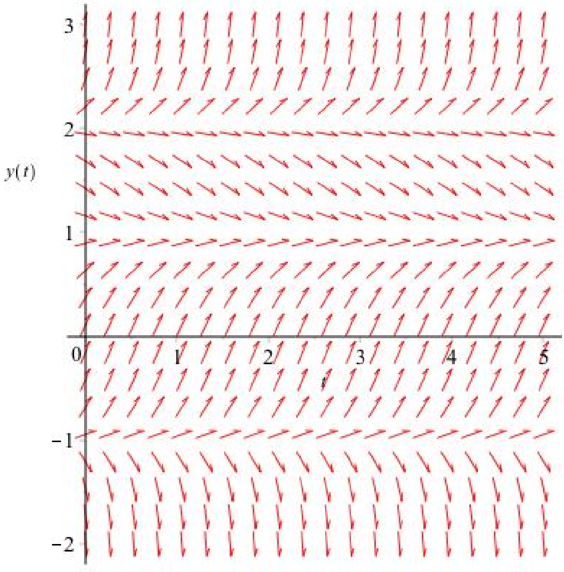
$$y_1 = \frac{2-\sqrt{7}}{3}, y_2 = \frac{2+\sqrt{7}}{3}$$



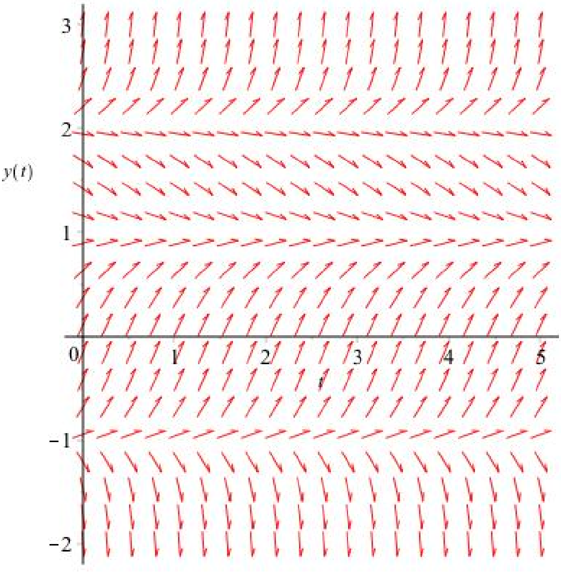
Since $y'(t) = f(y(t))$, we have $y''(t) = \frac{df}{dy}\frac{dy}{dt} = f''(y(t))y'(t)$

Interval	$y'(t)$	$y''(t)$	Behavior of Solution
$y < -1$	-	-	Decreasing Concave Down
$-1 < y < r_1$	$+$	$+$	Increasing, Concave Up
$r_1 < y < 1$	$+$	$-$	Increasing, Concave Down
$1 < y < y_2$	$-$	$+$	Decreasing, Concave Up
$y_2 < y < 2$	$-$	$-$	Decreasing, Concave Down
$y > 2$	$+$	$+$	Increasing, Concave Up

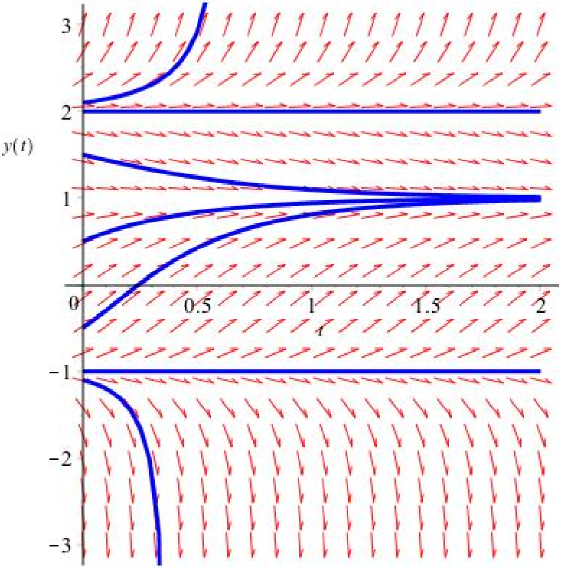
Direction Field for $y' = (y-1)(y+1)(y-2)$



Direction Field for $y' = (y^2 - 1)(y - 2)$



Some Integral Curves For $y' = (y^2 - 1)(y - 2)$



Some More Integral Curves For $y' = (y^2 - 1)(y - 2)$