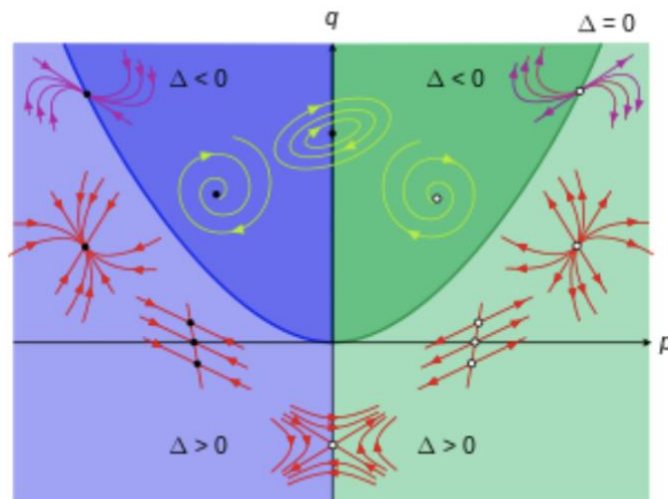




# MATH 226

## *Differential Equations*



$$\begin{aligned} \frac{dx}{dt} &= Ax + By & p &= A + D \\ \frac{dy}{dt} &= Cx + Dy & q &= AD - BC \\ & & \Delta &= p^2 - 4q \end{aligned}$$

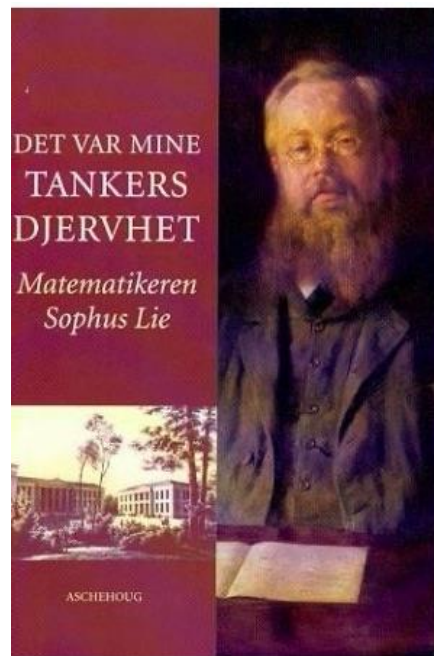
Spring 2025

*Michael Olinick*

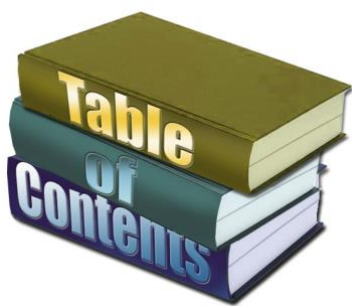


“Among all of the mathematical disciplines the theory of differential equations is the most important ... It furnishes the explanation of all those elementary manifestations of nature which involve time.”

### *Sophus Lie*



December 17, 1842 – February 18, 1899



Course Description/Syllabus

Tentative Course Outline

Schedule

Guide To Assignments

Assignment 0

Advice on Reading Mathematics Texts

On Studying and Learning Mathematics

On Problem Solving

What To Do For Tomorrow

## ***MATH 226: Differential Equations***

### **Course Description**

**Spring Term 2025**

***Generic Catalog Description:*** This course provides an introduction into ordinary differential equations (ODEs) with an emphasis on linear and nonlinear systems using analytical, qualitative, and numerical techniques. Topics will include separation of variables, integrating factors, eigenvalue method, linearization, bifurcation theory, and numerous applications. . In this course, we will introduce MATLAB programming skills and develop them through the semester. (MATH 0200 or by waiver) 3 hrs. lect./disc. **DED**

***More Specific Description for Spring 2025:*** This course provides a modern introduction to ordinary differential equations. We will emphasize analytic, geometric and numerical approaches to solving differential equations. We will also consider how differential equations can be used as powerful tools in modeling real world phenomena.

There will be an initial focus on *linear* differential equations. We will use extensively the tools of linear algebra to study such equations. After discussing first order equations and systems of first order equations, we will see how higher order equations can be analyzed.

We will then turn our attention to *nonlinear* differential equations, with an examination of autonomous systems and questions of stability. This unit will conclude with an introduction to the exciting new area of *chaos theory*. We will finish the course, as time permits, with an examination of some numerical methods to approximate the solutions of differential equations.

We will explore applications throughout by examples, problems, and projects.

***Learning Goals:*** Students will learn aspects of

- ***Methods of analysis and theory*** for solving differential equations analytically, as well as how to describe properties of solutions using theoretical concepts.
- ***Numerical and quantitative analysis***. Students will be able to identify when and how to implement numerical methods to solve differential equations using *Maple* or *MATLAB*.
- ***Graphical and qualitative analysis/representation*** of differential equations and solutions. Students will learn to create and interpret (by hand and by computer) direction fields, phase lines, phase planes, and plots of analytical and numerical solutions.
- ***Effective communication in mathematics*** by receiving regular feedback for written solutions, participating in group activities, and completing projects.

***Instructor:*** Michael Olinick, Office: Warner 202, Phone: 443-5559. Home telephone: 388-4290; email: [molinick@middlebury.edu](mailto:molinick@middlebury.edu). My usual office hours will be Monday, Wednesday and Friday from 10:40 AM to 1 PM. I would be happy to make an appointment to see you at other mutually convenient times.

***Meeting Times:*** Monday, Wednesday and Friday, 9:45 AM - 10:35 PM in Warner 010.

***Course Website:*** <http://s25.middlebury.edu/MATH0226A> or follow the link from my personal webpage <http://www.middlebury.edu/~molinick>.

**Computer Algebra System:** Mathematically oriented software such as *MATLAB*, *Maple*, and *Mathematica* give you an opportunity to investigate the ideas of multivariable calculus in ways not available to previous generations of students. Relatively simple commands can direct a computer to carry out complex calculations rapidly and without error. More importantly, you can create and carry out experiments to develop and test your own conjectures. The very powerful graphics capabilities of these applications provide you with strong tools to deepen your understanding of multivariable calculus through visualization of curves and surfaces. *MATLAB* is an especially powerful computational and visualization tool which is used extensively in scientific research and engineering applications. Intermediate and advanced required and elective courses in Middlebury's Applied Mathematics Track adopt *MATLAB* as the de facto programming tool. There will be a required introductory *MATLAB* workshop late in the first week of the term; you will be able to choose either a Thursday evening or Friday afternoon session.

**Prerequisites:** MATH 200 (Linear Algebra)

**Textbook:** James R. Brannan and William E. Boyce, *Elementary Differential Equations: An Introduction to Modern Methods and Applications*, Third Edition, Wiley: 2015. The text is available in several different formats: hardcover, loose leaf, or eTextbook. You can purchase new or used hardcover versions through the College Store; other versions are available from Amazon or other online vendors. A copy is also on reserve at Davis Family Library.

Your daily assignments will include a few pages of reading in the text. Be certain to read the book carefully (with pencil and paper, or occasionally *MATLAB* or *Maple*, close by!) Complete the relevant reading before coming to class and before tackling the exercises.

**Requirements:** There will be two midterm examinations and a final examination in addition to required homework assignments. The midterm examinations will be given in the evening to eliminate time pressure. Tentative dates for these tests are:

Wednesday, March 12

Wednesday, April 23

The College's Scheduling Officer has set 9 AM – Noon on Friday, May 16 as the time and date for the final exam for our course.

**Homework:** Mathematics is not a spectator sport! You must be a participant. The only effective way to learn mathematics is to do mathematics. We may occasionally assign some challenging problems which everyone may not be able to solve. You should, however, make an honest attempt at every problem.

You may use your notes, textbooks, calculators, and any computer software you have available (including *MATLAB*) to assist with the homework. Bear in mind, however, that none of these will be permitted during examinations.

I encourage you to talk to each other about the *Practice Exercises* and *Feedback Problems*. **The final write up must be done alone.** You should not have access to the assignment of a colleague while writing up your own. Warning: The College deals quite severely with cases of plagiarism, cheating, or other forms of academic dishonesty.

Review Middlebury's policy on academic honesty at [http://www.middlebury.edu/about/handbook/student\\_policies/Academic\\_Disciplinary\\_Policies](http://www.middlebury.edu/about/handbook/student_policies/Academic_Disciplinary_Policies)

Homework must be done neatly and legibly. Shoddy work will not be accepted for grading. Staple your assignments! There will not be a great deal of partial credit given for obviously incorrect answers. You should check your results where possible or at least examine them to see whether they are plausible.

**Practice Problems:** These problems are designed to hone the skills we will learn in this course. You can complete most problems with pencil and paper, while some require software. We expect you to complete all these problems but not all are graded or collected.

**Feedback Problems:** These are subsets of the practice problem sets that will be collected and graded for feedback.

**Work Together, Write Alone:** We strongly encouraged you to work together in pairs or small groups. However, all final drafts of feedback problems should be completed separately and in your own words.

**Take Pride In Your Work.** Feedback problems are expected to be neat, organized, legible, and stapled. Poorly written or messy work is not acceptable.

**Submit All Work On Time.** No late work will be accepted. Because we will distribute solution sheets to assigned work on the day it is due, we can not accept late papers. You should start the assignments early and work on them every day.

**Important Thought:** One of the essential characteristics of college life that distinguishes it from secondary school is the increased responsibility placed on you for your own education. Most of what you will learn will not be told to you by a teacher inside a classroom. Even if our model of you were an empty vessel waiting passively to be filled with information and wisdom, there would not be time enough in our daily meetings to present and explain it all.

We see you, more appropriately, as an *active* learner ready to confront aggressively the often times subtle and difficult ideas our mathematics courses contain. You will need to listen and to read carefully, to master concepts by wrestling with numerous examples and problems and individual/team projects, and frequently to ask thoughtful questions or make suggestions/conjectures about the course material.

**Grades:** Grades in the course will be based on the two midterm examinations, feedback problems, small group projects and the comprehensive final exam. The relative weights of the various components of the course are roughly as follows:

Examination 1:	25%
Examination 2:	25%
Three Projects:	20%
Final Examination:	30%

**Policy on Generative AI:** Please see our [Policy on AI Usage](#) Any use of generative AI tools will be treated as a violation of Middlebury's Honor Code.



**Help:** Please see me immediately if you have any difficulties with this course. Do not hesitate to utilize office hours. I welcome questions of any sort, including questions on assignments not yet handed in. In addition, I always appreciate your opinions, comments and suggestions concerning the course.

Halima Dieye will be the homework grader for the course. The College will offer drop-in tutoring sessions, most likely on Sunday and Wednesday evenings from 7:30 to 9:30 PM.

Students may also obtain many different forms of assistance from the Center for Teaching, Learning and Research (<http://www.middlebury.edu/academics/resources/ctlr>) in the Davis Family Library, the Q-Center (Armstrong Quantitative Center) in BiHall, and the Disability Resources Center (<http://www.middlebury.edu/office/disability-resource-center>) . I encourage you to investigate the services they offer.

**Accommodations:** Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through Student Accessibility Services. Please contact Jodi Litchfield or Peter Ploegman through the Disability Resource Center for more information: All discussions will remain confidential.

### ***Expectations***

- **Be There:** Attend all lectures, arriving on time, and staying for the duration of the class period.
- **Be Prepared:** We expect students to complete assigned readings prior to the class. Reading a mathematics text requires a pencil and paper. Do not stress about understanding every detail you read, but focus on getting a general picture of the topics discussed, and understanding most of the examples. Completing these readings will enhance the lecture experience for all of us.
- **Be Present:** Plan to participate in lectures by both asking and answering questions, as well as by taking part in discussions and group activities.
- **Be Proactive** in your understanding. Complete assignments regularly. Ask questions as they come to you. Attend office hours for clarification the moment you run into trouble.
- **Be Respectful** of yourself, your classmates, your instructor, and our classroom. This is our shared experience, and we are all partially responsible for ensuring a successful semester as a productive, welcoming, and stimulating class environment.
- **Be Honorable:** Students are expected to follow the Honor Code for all activities in this course. Expectations for feedback assignments, exams, and projects will be discussed explicitly in advance during class and students will be required to write/sign the honor pledge on larger assignments.

**A Final Word: There is a lot of exciting mathematical material in this course. Have fun with it!**



## Generative AI in Our Class



Here is an image depicting Isaac Newton and Gottfried Wilhelm Leibniz in a heated feud over the creation of calculus. The scene captures the tension between the two historical figures, surrounded by symbols of their mathematical contributions. (Generated by ChatGPT4)

**AI Use Policy for Our Course:** This policy aims to foster a productive and ethical learning environment where students can use AI to support their education while maintaining the integrity of their work

*1. Purpose of AI Tools:* - AI tools can be valuable for enhancing learning and understanding complex calculus concepts. I encourage students to use AI for study aids, explanations, and practice problems to supplement their learning.

### *2. Permissible Uses*

- **Study Assistance:** Students may use AI tools such as calculators, solvers, or educational platforms to check your work, understand solutions, and explore additional problems.
- **Concept Clarification:** AI can be used to clarify concepts, get alternative explanations, or see step-by-step solutions to practice problems.
- **Preparation for Exams:** Students may use AI to generate practice problems, review materials, and study guides.

### *3. Prohibited Uses:*

- **Assignment Completion:** Students are prohibited from using AI to complete assignments, quizzes, or exams. This includes using AI to solve problems directly, generate answers, or perform any task that contributes to a graded assessment.
- **Plagiarism:** Using AI to generate written content (*e.g.*, essays, explanations) that is submitted as original work without proper attribution is considered plagiarism and will be treated as an

academic integrity violation. (Along those lines...I used ChatGPT 4 to write 90% of this AI policy.)

- Communication During Exams: AI tools must not be used during exams or quizzes unless explicitly allowed. This includes AI-powered calculators, chatbots, or any online resources.

4. *Transparency and Attribution*:

- When AI tools are used to aid in understanding or problem-solving, students should acknowledge the use of these tools. This includes citing the tool in written work if it contributes significantly to a solution or explanation.

5. *Consequences of Misuse*:

- Misuse of AI tools, particularly in ways that violate academic integrity, will result in disciplinary action according to Middlebury's policies. Consequences may include receiving a zero on the assignment, failing the course, or further disciplinary measures.

6. *Encouragement of Ethical Use*:

- I encourage students to use AI ethically to enhance your learning experience. The goal is to use these tools to develop a deeper understanding of calculus, not to bypass the learning process.

7. *Instructor's Discretion*:

- I reserve the right to modify this policy as needed and will communicate any changes to the class promptly. Any use of AI tools not explicitly covered by this policy should be discussed with me.

**Citation for my use of AI to generate this section**

"Write a policy on the use of AI for my calculus course" prompt. *ChatGPT 4.0*, OpenAI, August 27, 2024, [chat.openai.com/chat](https://chat.openai.com/chat); revised February 1, 2025.

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# MATH 226 Spring 2025

## *Tentative Course Outline*

### **Chapter 1: Review and Introduction** (2 weeks)

- Integration Review
- Mathematical Models and Solutions
- Qualitative Methods: Phase Lines and Direction Fields
- Definitions, Classification, and Terminology

### **First Order Differential Equations** (2½ weeks)

- Separable Equations
- Linear Equations: Method of Integrating Factors
- Modeling with First Order Equations
- Differences Between Linear and Nonlinear Equations
- Autonomous Equations and Population Dynamics

### **Systems of Two First Order Equations** (2½ weeks)

- Matrices and Linear Algebra
- Systems of Two First Order Linear Differential Equations
- Homogeneous Linear Systems with Constant Coefficients
- Complex Eigenvalues
- Repeated Eigenvalues

### **Systems of First Order Linear Equations** (3 weeks)

- Definitions and Examples
- Basic Theory of First Order Linear Systems
- Homogeneous Linear Systems with Constant Coefficients
- Nondefective Matrices with Complex Eigenvalues
- Fundamental Matrices and the Exponential of a Matrix
- Nonhomogeneous Linear Systems

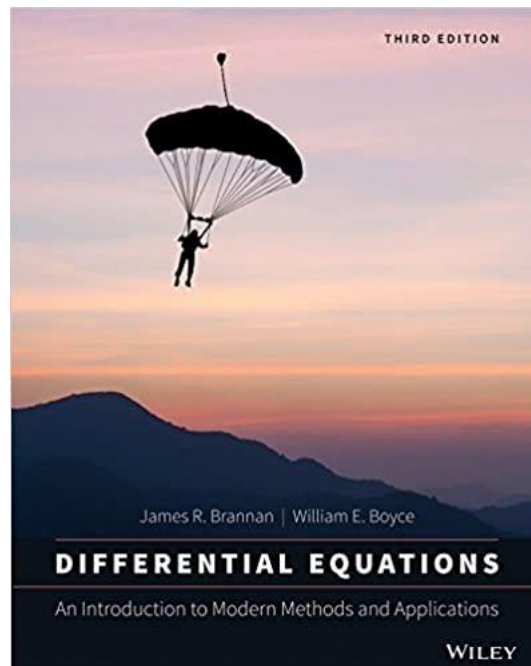
### **Nonlinear Differential Equations and Stability** (2 -3 weeks)

- Autonomous Systems and Stability
- Almost Linear Systems
- Competing Species
- Predator-Prey Equations
- Periodic Solutions and Limit Cycles
- Chaos and Strange Attractors: The Lorenz Equations

**MATH 226 Differential Equations**  
**Guide To Assignments for Spring 2025**

Most assignments will be due in class on Monday or Friday. Homework will generally focus on material introduced in the previous class meetings. Begin the new assignment as soon after class as practical so that you will have two days to complete it. Review your class notes and read the assigned sections first, and then work the problems. We encourage you to study together and wrestle with hard problems, but submitted solutions should be your own work. Come to class with well-prepared and specific questions on the assignment.

Experience shows that doing the assignments regularly and carefully is the key to doing well in calculus. You should expect to spend at least two hours on homework for every hour in class, and an average of 8 to 10 hours total each week including your review and careful reading. Most of the readings and exercises will come from our text, *Differential Equations: An Introduction to Modern Methods and Applications* by James Brennan and William Boyce.



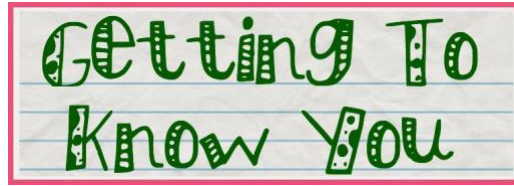
The reading is an important supplement to what goes on in class. It will cover some examples and methods you need and for which you will be held responsible but which we will not have time to cover in class. Read carefully with a pencil and pad next to you. Work out details of calculations you don't understand.

# MATH 226: *Differential Equations*

## Tentative Schedule for Spring Term 2025

pte	Monday	Tuesday	Wednesday	Thursday	Friday
February 10	<i>FIRST DAY OF CLASSES</i>		Assignment 0	<b>MATLAB Workshop 7:30m- 9:30</b>	Assignment 1 <b>MATLAB Workshop 2:30m- 4:30</b>
February 17	Assignment 2				Assignment 3
February 24	Assignment 4				Assignment 5
March 3	Assignment 6				Assignment 7
March 10	Assignment 8		<b>Exam 1</b>		<i>Assignment 8</i>
March 17	<i>MID TERM RECESS</i>	<i>MID TERM RECESS</i>	<i>MID TERM RECESS</i>	<i>MID TERM RECESS</i>	<i>MID TERM RECESS</i>
March 24	Assignment 9				Assignment 10
March 31	Assignment 11				Assignment 12
April 7	Assignment 13				Assignment 14
April 14	Assignment 15		<b>Exam 2</b>		Assignment 16
April 21	Assignment 17				Assignment 18
April 28	Assignment 19				Assignment 20
May 5	Assignment 21				Assignment 22
May 12	Assignment 23				<b>Final Exam 9 – Noon</b>

Assignment 0  
(Adapted from *Homework 0* by Alex Lyford)  
**Due: Wednesday, February 12**



### Reading

Read carefully through the [Integration Review](#).

### Writing

You may submit an electronic copy of this assignment to me ([molinick@middlebury.edu](mailto:molinick@middlebury.edu)) with the subject line: **MATH 226 Assignment 0** or print it out and bring it to Wednesday's class. **Make sure you include your name at the top of the document.**

Your task is to create a document describing yourself, your goals, and what you hope to get out of our Differential Equations class. Please provide your name at the top of the first page along with your major or likely major and your anticipated graduate date.

Start with an autobiographical statement about yourself that will help me to get to know a little about you. Where did you grow up? Why did you come to Middlebury? What are your likes and dislikes? Do you have any hobbies that you do regularly? Do you have a major extracurricular activity such as athletics, theatre or *The Campus*?

After the biographical statement tell me about your mathematical, statistical, and computer programming background. What did you like about previous mathematics, statistics, and/or programming classes? What did you dislike? What aspects did you find easy? What aspects did you find challenging?

The next part should discuss your plans for the remainder of your time in college, and what you hope to do after you graduate. Is more schooling the next step, or do you plan to get a job? It's okay to not have any idea what you want to do after graduation, but list some possibilities so that I can better tailor the materials in class to your potential career opportunities.

Finally tell me about your thoughts and expectations for this class. What are you hoping and/or expecting to learn? What do you think the challenges of this course might be? What, if anything, have you heard about this course from your peers? What expectations do you have of me? Feel free to also discuss anything I've failed to ask here!

## **MATLAB**

Install *MATLAB* (version R2024b) on your computer using this link

(<https://www.mathworks.com/academia/tah-portal/middlebury-college-31511644.html#get>)

Use your Middlebury email address and password to sign i

### ***Advice on Reading Your Mathematics Textbooks***

[There are many excellent essays online with advice on how to read effectively a mathematics book. Here is an adaption of one of them. I haven't been able to track down the original author's name to give full credit.]

Reading the textbook is important for succeeding academically, and this is also true in your mathematics classes. However, reading mathematics is different from other types of reading. Getting the most out of a mathematics textbook will require more than just skimming through the pages. Below are some tips for helping you get the most from your mathematics text.

- **Focus on concepts, not exercises.** The most important material in a mathematics textbook is found in the prose, not in the exercises at the end of the section. In the past, you may have opened your mathematics book only when doing problem sets and exercises (looking at the rest of the book only for examples which mirror the current assigned homework). You must rid yourself of this bad habit now. Instead, set aside time to read the text when you are not working on a homework assignment. This will enable you to truly focus on the mathematical concepts at hand.

There are an infinite number of types of mathematics problems, so there is no way to learn every single problem-solving technique. Mathematics is about ideas. The mathematics problems that you are assigned are expressions of these ideas. If you can learn the key concepts, you will be able to solve any type of problem (including ones you have never seen before) that involves those concepts.

- **Read the text more than once.** You cannot read mathematics in the same way as you would read a newspaper or a novel. Many of the ideas presented in a typical college mathematics course have confounded brilliant minds in centuries past. So it is not unexpected that you may have difficulty learning these same ideas if you quickly scan through the reading assignments only once. You should expect to go through the each reading assignment several times before you can gain a full understanding of the material.

- **When reading through for the first time, look for the big ideas.** The first time you read through a chapter of the textbook, you should be thinking to yourself: “What is the main point of the chapter?” Look for the big picture. The details are important, but you need to be aware of the forest first before focusing on the trees.

- **The second time through, fill in details.** After you get the big picture, you should then look at the details. Take some time to think about each of the definitions, theorems, and formulas you encounter (more on this later).

- **Read with paper and pen.** As you are reading through the text, you should be writing notes and verifying any parts of which you are skeptical. Check any calculations. Rewrite definitions and theorems in your own words.



See if you can come up with your own examples. Ask yourself about special cases of the theorems you read.

- **Read the narrative.** There is a story to be told in mathematics. What is the progression of ideas being told? Don't just skip to the formulas and examples, but instead follow the development of the ideas and concepts presented.
- **Study the examples.** What points do each of the examples illustrate? Some examples are extreme cases. Other examples are supposed to illustrate "typical" situations.
- **Read the pictures.** There are good reasons for the many pictures and graphs in mathematics texts. You should be asking yourself what features of the picture are important to the key concepts. Focus on how each picture illustrates a particular idea.
- **Learn the vocabulary and the language.** Pay attention to definitions and what they mean. Mathematics language is very precise, and a word in a mathematical context may have a different meaning than when it is used in everyday conversation. In mathematics, great care is taken to explicitly and precisely define the notions being considered. In addition, mathematical definitions and language are crafted in such a way to convey sophisticated notions in as simple and concise a manner as possible.
- **Learn the theorems and what they mean.** Theorems are vital bricks to building mathematical knowledge. When you see a theorem in a mathematics text, look at it very closely. What does it say? What are its hypotheses? What implications does it have? Are there special cases you should be aware of? Can you think of examples to which the theorem applies? Can you think of examples that do not satisfy the hypotheses and the conclusion of the theorem?
- **Use the index and the appendices.** Know what every word means. Make sure that you understand all of the words and ideas. If there is a particular word which you do not know (or which you want to know better), look it up. Use the table of contents or the index to help you.
- **Make a note of things you do not understand, and ask for help afterwards.** Even after following all of the above advice, you might still find some of the ideas confusing. That is to be expected; material such as this is often hard to internalize when one first encounters it. If there is something that you do not understand, make a note of it. Write down any questions you may have. You then can bring up these issues with your instructor or a classmate.

## ON STUDYING AND LEARNING MATHEMATICS

Past students have found that some ways are far more effective than others in studying and learning mathematics. Here are some suggestions and pointers that may help you in budgeting the time you can devote to mathematics, preparing for examinations, and learning and understanding the material in a way that promotes long-range retention:

1. Do all reading assignments actively. Keep a pencil and scratch paper at hand. Mark up the pages of the book. Write in any questions you may have. Verify examples given by writing out the details yourself.
2. Plan to do all reading assignments several times. In mathematics courses, reading assignments are seldom more than a few pages long. They often contain, however, subtle ideas which require repeated study before they are mastered. You should read the appropriate section of the text before the class in which it will be discussed, read it again before beginning the homework assignment, and read it a third time after you have completed the homework.
3. Follow the advice in (1) above when reviewing your lecture notes. You should try to go over your lecture notes as soon as possible after the class session has ended. Definitely review the notes before attempting the homework.
4. Do all homework sets on time. Don't let yourself fall behind. If you have difficulty with a problem, especially one that is more theoretical, do the following:
  - (a) Write out the relevant definitions and results. It may now be a small step to complete the problem.
  - (b) Ask whether you can think of a simpler but related problem, and tackle that one first. Is there a special case of the general result? Do you know how to solve the problem in this special case? This approach usually provides insight for attacking the original problem.
5. Do not spend hours sitting still, thinking, reading, studying and reviewing problem solutions! While these approaches may be helpful for other courses and some time should be spent on these activities in mathematics courses, there are more productive paths to learning in mathematics. Spend your time writing out solutions to new problems, deriving relationships, writing down clear definitions, and outlining the steps of a proof. These activities provide a better way to prepare for an examination.
6. Pay a great deal of attention to definitions. Write them out yourself and think about them. Write out examples that do and do not satisfy the definitions. Ask yourself how the definition says something different from its intended meaning if the order of the words is shifted.
7. Begin reviewing for examinations a week early. Use small chunks of time. Tackle those topics you have found difficult; with hindsight they are often easier. Do NOT plan on spending a whole day of study just before an exam. This is almost always an inefficient way to budget your time.
8. Review solutions for homework problems as soon as you get them, and write up (for your own enlightenment) those problems which caused you difficulty.
9. Write down questions that arise as you go along. Bring them with you to class, to review sessions, and to your instructor's office hours.

## ON PROBLEM SOLVING

A major part of your time in Multivariable Calculus and other courses is devoted to solving problems. It is worth your while to develop sound techniques. Here are a few suggestions.

Think. Before plunging into a problem, take a moment to think. Read the problem again. Think about it. What are its essential features? Have you seen a problem like it before? What techniques are needed?

Try to make a rough estimate of the answer. It will help you understand the problem and will serve as a check against unreasonable answers. A car will not go 1,000 miles in 3 hours; a weight dropped from 10,000 feet will not hit the earth at 5 mph; the volume of a tank is not -275 gal.

Examine the data. Be sure you understand what is given. Translate the data into mathematical language. Whenever possible, make a clear diagram and label it accurately. Place axes to simplify computations. If you get stuck, check that you are using all the data.

### **Avoid sloppiness.**

(a) *Avoid sloppiness in language.* Mathematics is written in English sentences. A typical mathematical sentence is " $y = 4x + 1$ ." The equal sign  $=$  is the verb in this sentence; it means *equals* or *is equal to*. The equal sign is not to be used in place of *and*, nor as a punctuation mark.

Quantities on opposite sides of an equal sign must be equal.

Use short simple sentences. Avoid pronouns such as "it" and "which". Give names and use them. Consider the following example.

"To find the minimum of it, differentiate it and set it equal to zero, then solve it which if you substitute it, it is the minimum."

Better: "To find the minimum of  $f(x)$ , set its derivative  $f'(x)$  equal to zero. Let  $x_0$  be the solution of the resulting equation. Then  $f(x_0)$  is the minimum value of  $f(x)$ ."

(b) *Avoid sloppiness in computation.* Do calculations in a sequence of neat, orderly steps. Include all steps except utterly trivial ones. This will help eliminate errors, or at least make errors easier to find. Check any numbers used; be sure that you have not dropped a minus sign or transposed digits.

(c) *Avoid sloppiness in units.* If you start out measuring in feet, all lengths must be in feet, all areas in square feet, and all volumes in cubic feet. Do not mix feet and acres, seconds and years.

(d) *Avoid sloppiness in the answer.* Be sure to answer the question that is asked. If the problem asks for the maximum value of  $f(x)$ , the answer is not the point where the maximum occurs. If the problem asks for a formula, the answer is not a number.

EXAMPLE                      Find the minimum of  $f(x) = x^2 - 2x + 1$ .

Solution 1:

$$\begin{aligned} 2x - 2 \\ x = 1 \\ 1^2 - 2 \cdot 1 + 1 \\ 0 \end{aligned}$$

Unbearable. This is just a collection of marks on the paper. There is absolutely no indication of what these marks mean or of what they have to do with the problem. When you write, it is your responsibility to inform readers what you are doing. Assume they are intelligent, but not mind readers.

Solution 2:

$$\begin{aligned}\frac{df}{dx} &= 2x - 2 = 0 = 2x = 2 = x = 1 \\ &= f(x) = 1^2 - 2 \cdot 1 + 1 = 0.\end{aligned}$$

Poor. The equal sign is badly mauled. This solution contains such enlightening statements as " $0 = 2 = 1$ ," and it does not explain what the writer is doing.

Solution 3:

$$\frac{df}{dx} = 2x - 2 = 0, \quad 2x = 2, \quad x = 1.$$

This is better than Solution 2, but contains two errors. Error 1: The first statement, " $\frac{df}{dx} = 2x - 2 = 0$ ," muddles two separate steps. First the derivative is computed, then the derivative is equated to zero. Error 2: The solution is incomplete because it does not give what the problem asks for, the minimum value of  $f$ . Instead, it gives the point  $x$  at which the minimum is assumed.

Solution 4: The derivative of  $f$  is

$$f' = 2x - 2.$$

At a minimum,  $f' = 0$ . Hence

$$2x - 2 = 0, \quad x = 1.$$

The corresponding value of  $f$  is

$$f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

If  $x > 1$ , then  $f'(x) = 2(x-1) > 0$ , so  $f$  is increasing. If  $x < 1$ , then  $f'(x) = 2(x-1) < 0$ , so  $f$  is decreasing. Hence  $f$  is minimal at  $x = 1$ , and the minimum values of  $f$  is 0.

This solution is absolutely correct, but long. For homework assignments the following is satisfactory (check with your instructor):

Solution 5:

$$f'(x) = 2x - 2.$$

At min,  $f' = 0$ ,  $2x - 2 = 0$ ,  $x = 1$ . For  $x > 1$ ,  $f'(x) = 2(x-1) > 0$ ,  $f \square$ ; for  $x < 1$ ,  $f'(x) = 2(x-1) < 0$ ,  $f \square$ .

Hence  $x = 1$  yields min,

$$f_{\min} = f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

The next solution was submitted by a student who took a moment to think.

Solution 6:

$$f(x) = x^2 - 2x + 1 = (x-1)^2 \geq 0.$$

But

$$f(1) = (1-1)^2 = 0.$$

Hence the minimum value of  $f(x)$  is 0.

- Adapted from *A First Course in Calculus*  
by Flanders, Korfhage and Price

# MATH 226: Differential Equations

Spring 2025

## *What To Do By Tomorrow*

- 1) Read through the documents in this packet.
- 2) Access the course website <http://s25.middlbury.edu/MATH0226A> and check out some of the links.
- 3) Obtain a copy of our text and familiarize yourself with its structure.
- 4) Purchase a loose leaf binder to store the various handouts about the course that will be distributed, your class notes and the homework. You will accumulate a large number of loose sheets of paper from this course during the term; it's very helpful to keep them organized.
- 5) Begin work on Assignment 0, the assignment due to be turned in on Wednesday.
- 6) Initiate review of integration techniques.
- 7) **Don't hesitate to come in to see me if you need some help or have questions.**