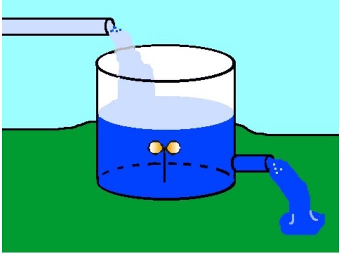
MATH 226: Differential Equations



Class 7: February 24, 2025

Notes on Assignment 4

Assignment 5

Modeling With First Order Differential Equations

Announcements

1. First Team Project
   - Tuesday: Requests for Team Members
   - Wednesday: Team Assignments
   - Week From Friday: Projects Due
2. Exam 1: Two Weeks From Wednesday Night

More Mathematicians of the Day

Method of Integrating Factors

Gottfried Leibniz
1646 – 1716
German

Leonhard Euler
1707 – 1783
Swiss

George Stokes
1819 – 1903
English

This Week

Modeling With First Order Differential Equations

Existence and Uniqueness Theorems

Population Growth Models

2. A tank initially contains 200 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 4 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

2. A tank initially contains 200 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 4 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

Let $Q(t)$ be the quantity (in grams) of salt in the tank at time t (in minutes)

What are units for $\frac{dQ}{dt}$?

grams/minute

Formulate a Differential Equation Who Solution Will be Q

$$\frac{dQ}{dt} = \text{ rate in} - \text{rate out}$$

Rate IN: Water containing γ grams/liter of salt flows in a rate of 4 liters per minute.

Rate in is:

$$\gamma \frac{grams}{liter} \times 4 \frac{liters}{minute} = 4\gamma \frac{grams}{minute}$$

Let $Q(t)$ be grams of salt in tank at t (in minutes)

$$\frac{dQ}{dt} = \text{ rate in} - \text{rate out}$$

Rate in is: $= 4\gamma \frac{grams}{minute}$

$$\frac{dQ}{dt} = 4\gamma - \text{ rate out}$$

Rate OUT:

$$\frac{Q}{200}\frac{grams}{liter} \times 4\frac{liters}{minute} = \frac{Q}{50}\frac{grams}{minute}$$

Thus $\frac{dQ}{dt} = 4\gamma - \frac{Q}{50}$

Let $Q(t)$ be grams of salt in tank at t (in minutes)

$$\frac{dQ}{dt} = 4\gamma - \frac{Q}{50}$$

$$\frac{dQ}{dt} + \frac{Q}{50} = 4\gamma$$

The solution is : $Q(t) = 200\gamma + Ce^{-t/50}$
Using $Q(0) = 0$, we get $C = -200\gamma$
Thus $Q(t) = 200\gamma(1 - e^{-t/50})$. As $t \to \infty, Q(t) \to 200\gamma$.

7. An outdoor swimming pool loses 0.05% of its water volume every day it is in use, due to losses from evaporation and from excited swimmers who splash water. A system is available to continually replace water at a rate of G gallons per day of use.

(a) Find an expression, in terms of G, for the equilibrium volume of the pool. Sketch a few graphs for the volume $V(t)$, including all possible types of solutions.

(b) If the pool volume is initially 1% above its equilibrium value, find an expression for $V(t)$.

(c) What is the replacement rate G required to maintain 12,000 gal of water in the pool?