Brannan, Boyce:
Differential Equations: An Introduction to Modern Methods and Applications, 3rd Edi
Chapter 9

CHAPTER NINE

Series Solutions of Second Order Linear Equations

The general solution of a linear second order equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

is

$$y = c_1y_1(x) + c_2y_2(x),$$

where y_1 and y_2 are a fundamental set of solutions of the differential equation. So far, we have given a systematic procedure for constructing fundamental solutions only if the equation has constant coefficients:

$$ay'' + by' + cy = 0.$$

Furthermore y_1 and y_2 can be expressed in closed form in terms of elementary functions.

To deal with equations that have general nonconstant coefficients, it is necessary to consider alternative solution techniques. For some applications we may find that approximations using an initial value problem solver are satisfactory for our needs. However there are some variable coefficient equations that frequently recur in applications, and it is either convenient or necessary to represent their solutions by infinite series. For instance, a solution of

$$x^2y'' + xy' + x^2y = 0, \qquad x > 0,$$